TwisterForge: controllable and efficient animation of virtual tornadoes

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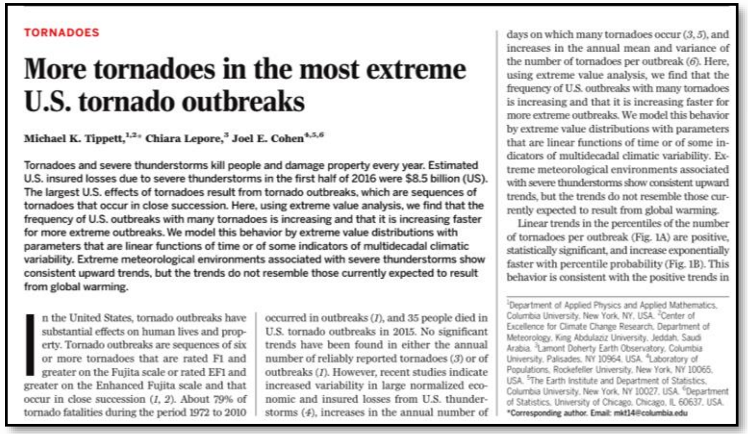
TORNADO OUTBREAKS



TORNADOES

More tornadoes in the most extreme U.S. tornado outbreaks

Michael K. Tippett,[1,2]* Chiara Lepore,[3] Joel E. Cohen[4,5,6]

Tornadoes and severe thunderstorms kill people and damage property every year. Estimated U.S. insured losses due to severe thunderstorms in the first half of 2016 were $8.5 billion (US). The largest U.S. effects of tornadoes result from tornado outbreaks, which are sequences of tornadoes that occur in close succession. Here, using extreme value analysis, we find that the frequency of U.S. outbreaks with many tornadoes is increasing and that it is increasing faster for more extreme outbreaks. We model this behavior by extreme value distributions with parameters that are linear functions of time or of some indicators of multidecadal climatic variability. Extreme meteorological environments associated with severe thunderstorms show consistent upward trends, but the trends do not resemble those currently expected to result from global warming.

In the United States, tornado outbreaks have substantial effects on human lives and property. Tornado outbreaks are sequences of six or more tornadoes that are rated F1 and greater on the Fujita scale or rated EF1 and greater on the Enhanced Fujita scale and that occur in close succession (*1*, *2*). About 79% of tornado fatalities during the period 1972 to 2010 occurred in outbreaks (*1*), and 35 people died in U.S. tornado outbreaks in 2015. No significant trends have been found in either the annual number of reliably reported tornadoes (*3*) or of outbreaks (*1*). However, recent studies indicate increased variability in large normalized economic and insured losses from U.S. thunderstorms (*4*), increases in the annual number of days on which many tornadoes occur (*3*, *5*), and increases in the annual mean and variance of the number of tornadoes per outbreak (*6*). Here, using extreme value analysis, we find that the frequency of U.S. outbreaks with many tornadoes is increasing and that it is increasing faster for more extreme outbreaks. We model this behavior by extreme value distributions with parameters that are linear functions of time or of some indicators of multidecadal climatic variability. Extreme meteorological environments associated with severe thunderstorms show consistent upward trends, but the trends do not resemble those currently expected to result from global warming.

Linear trends in the percentiles of the number of tornadoes per outbreak (Fig. 1A) are positive, statistically significant, and increase exponentially faster with percentile probability (Fig. 1B). This behavior is consistent with the positive trends in

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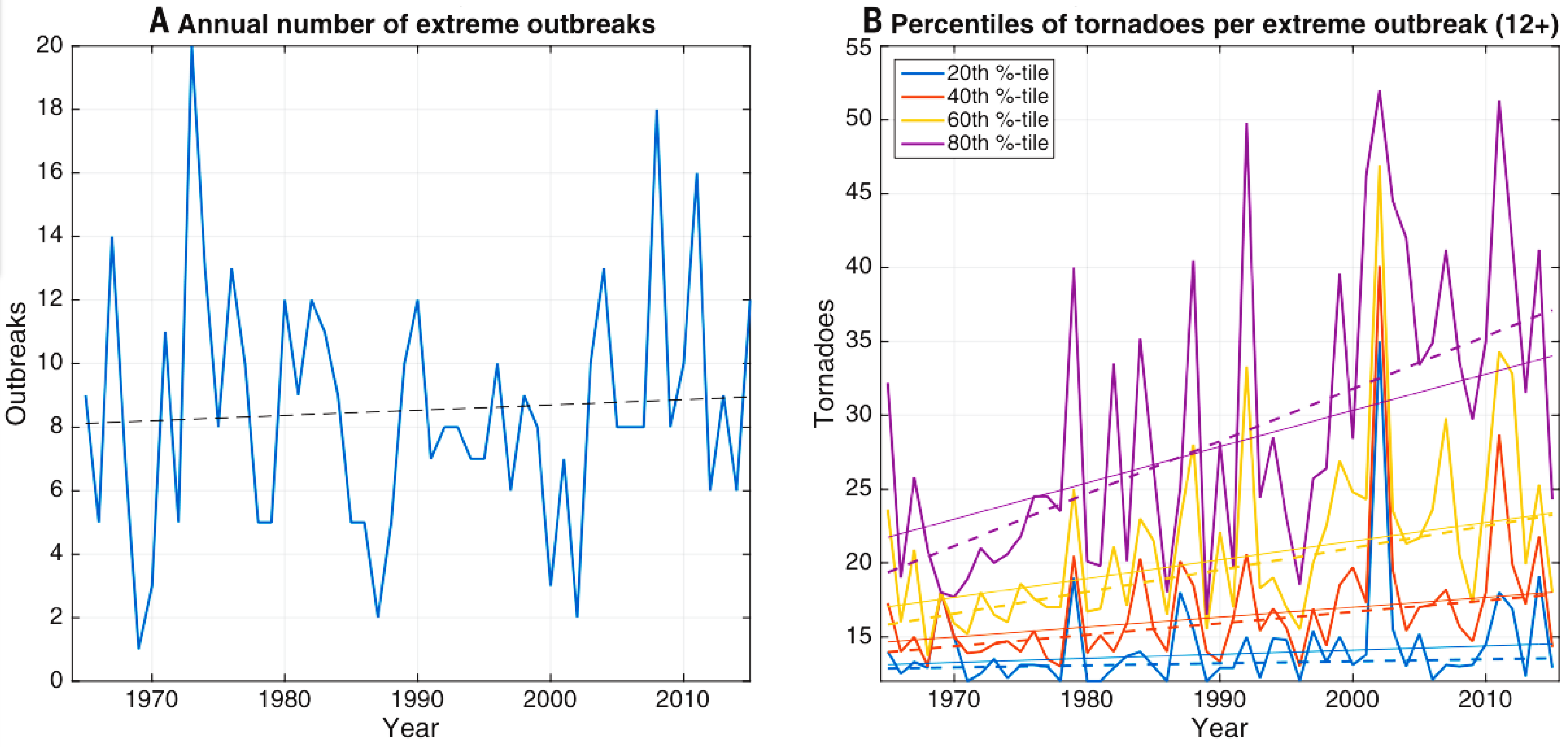
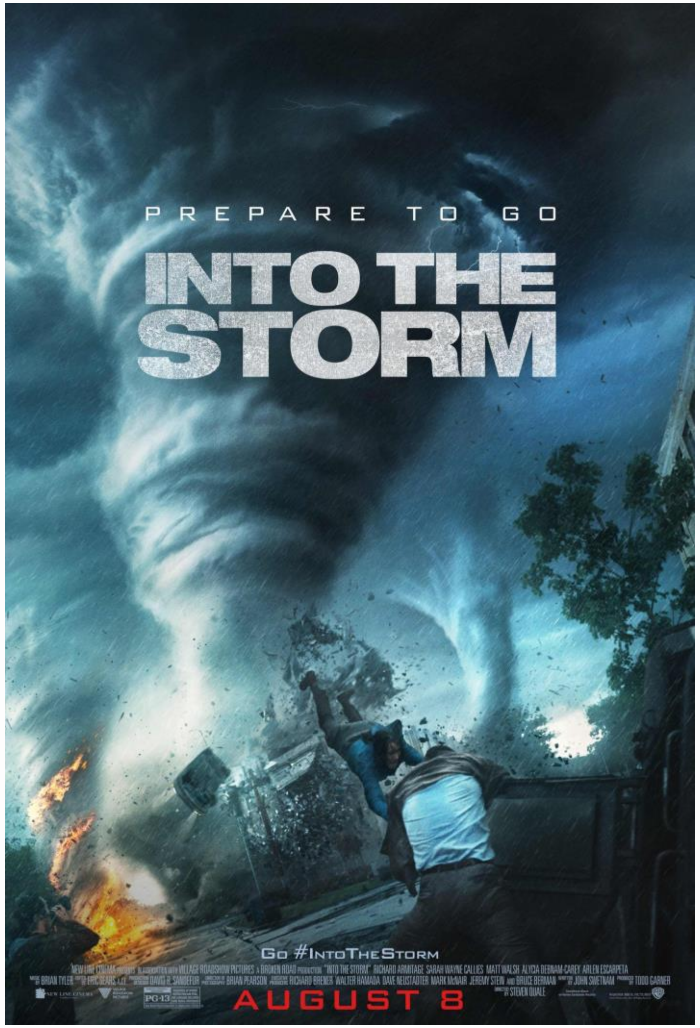
[Tippett et al. 2016], Science

The frequency of U.S. outbreaks with many tornadoes is increasing and it is increasing faster for more extreme outbreaks.



TORNADO ANIMATION FOR ENTERTAINMENT



Movies

Video games





PHYSICALLY BASED NUMERICAL SIMULATION

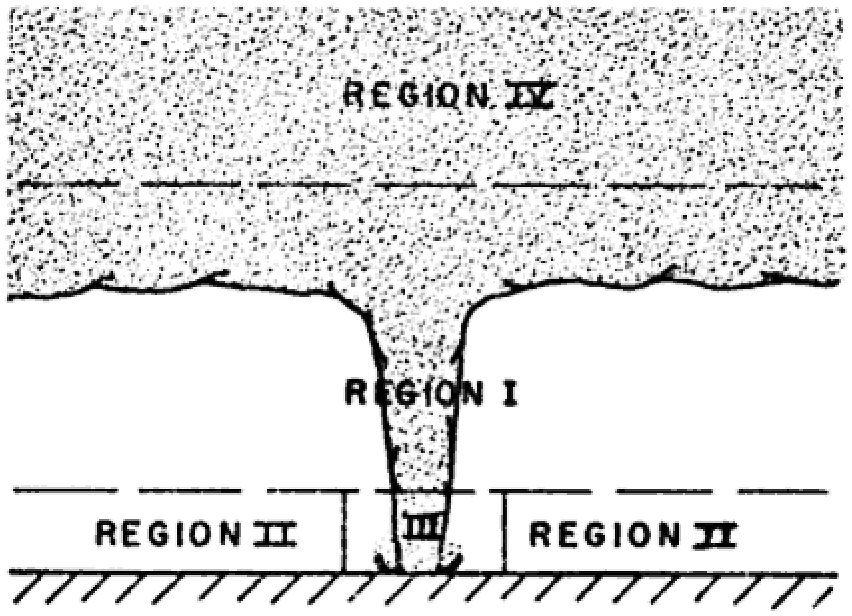
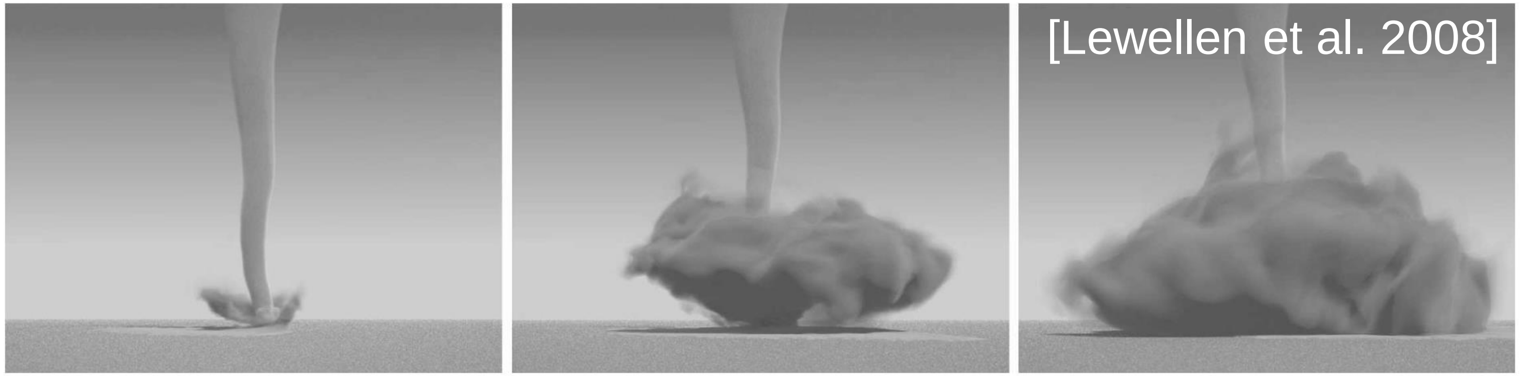


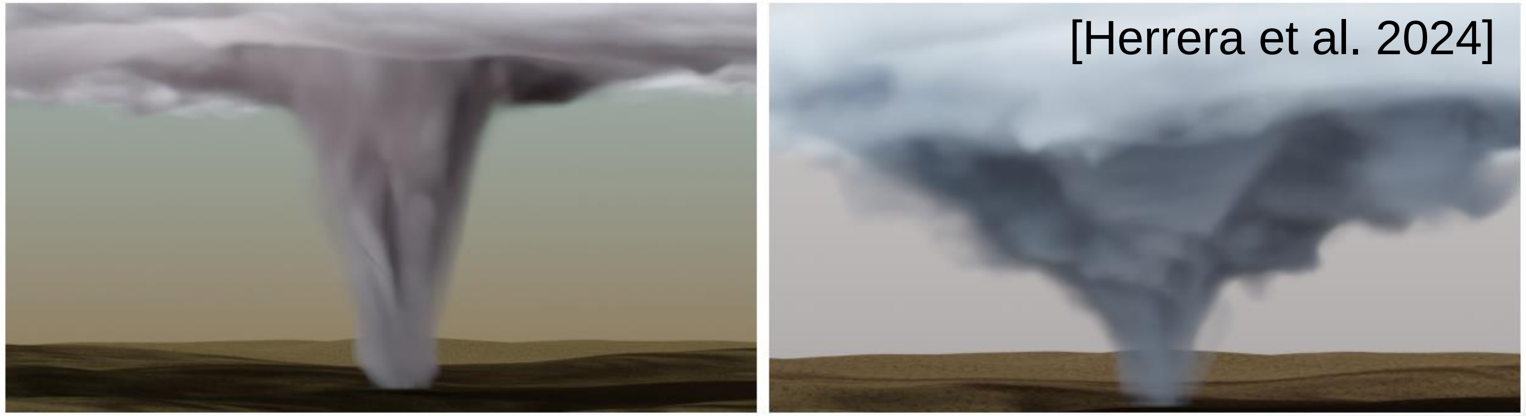
Fig. 1. Sketch showing the division of the tornado into four separate regions.

[Lewellen 1976]



[Lewellen et al. 2008]

[Liu et al. 2008]



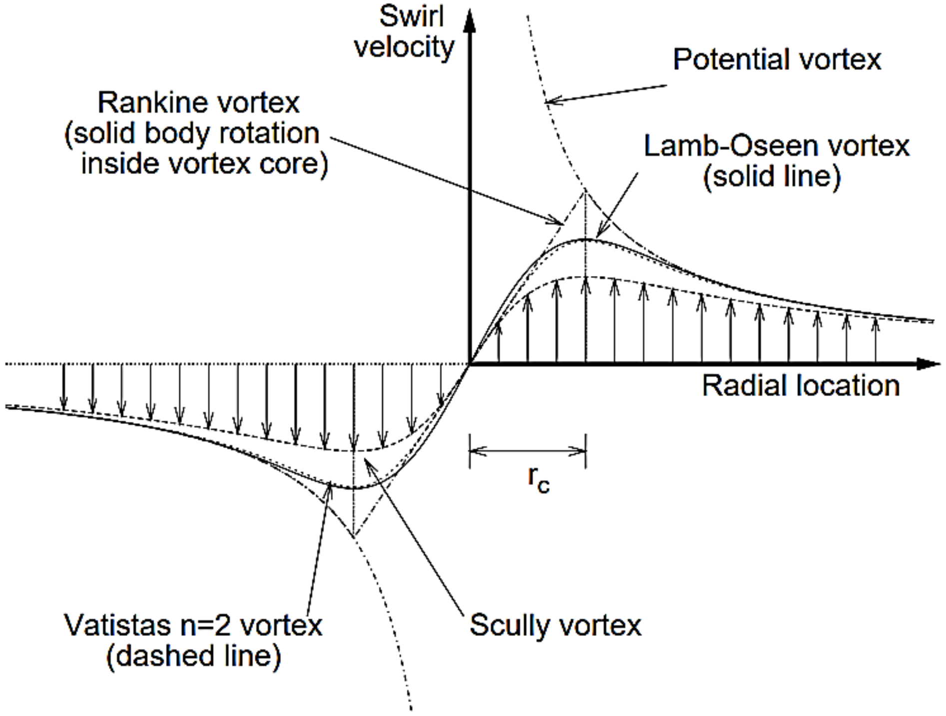
[Herrera et al. 2024]

ANALYTICAL VORTEX SOLUTIONS

- Simplify the 3D NS equation
  - Assuming helical symmetry of the flow

Axisymmetric steady flow

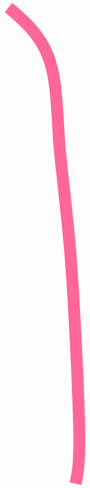
$$\begin{cases} v_r \dfrac{\partial v_r}{\partial r} + v_z \dfrac{\partial v_r}{\partial z} - \dfrac{v_\theta^2}{r} = \\ \qquad -\dfrac{1}{\rho}\dfrac{\partial p}{\partial r} + \mu\left(\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial v_r}{\partial r}\right) + \dfrac{\partial^2 v_r}{\partial z^2} - \dfrac{v_r}{r^2}\right), \\ v_r \dfrac{\partial v_\theta}{\partial r} + v_z \dfrac{\partial v_\theta}{\partial z} + \dfrac{v_\theta v_r}{r} = \mu\left(\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u_\theta}{\partial r}\right) - \dfrac{v_\theta}{r^2}\right), \\ v_r \dfrac{\partial v_z}{\partial r} + v_z \dfrac{\partial v_z}{\partial z} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z} + \mu\left(\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u_z}{\partial r}\right)\right), \end{cases}$$

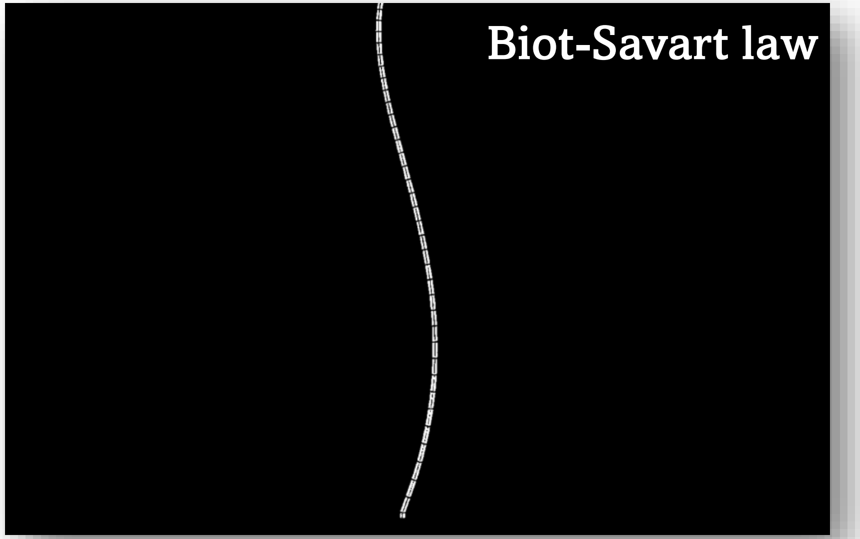


OUR APPROACH

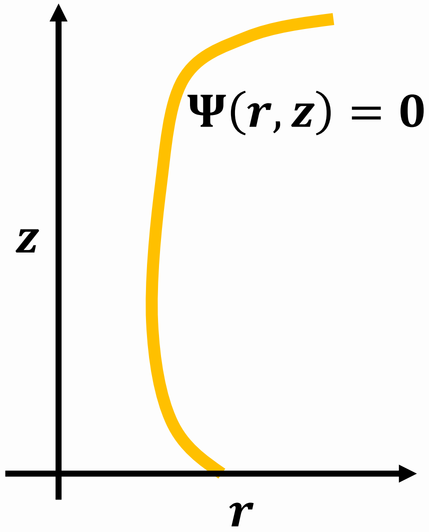
A core-and-funnel model

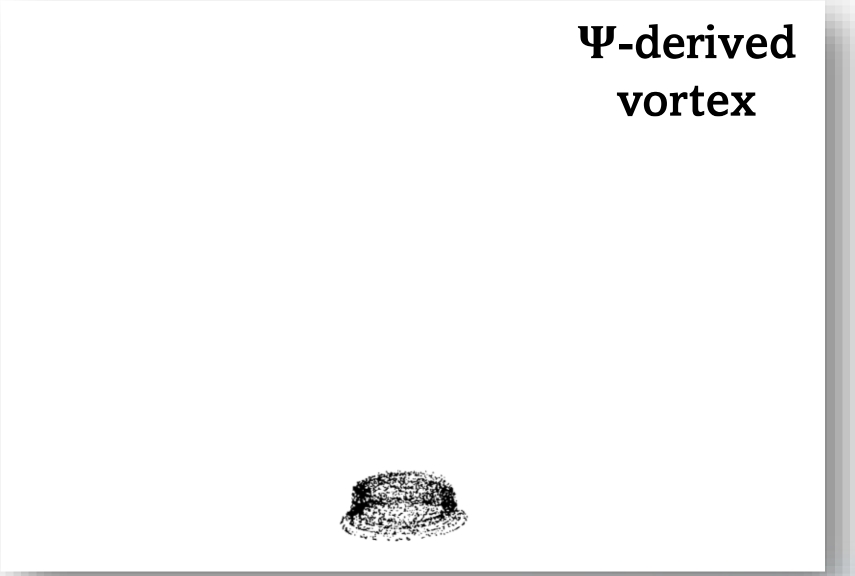
Vortex filament



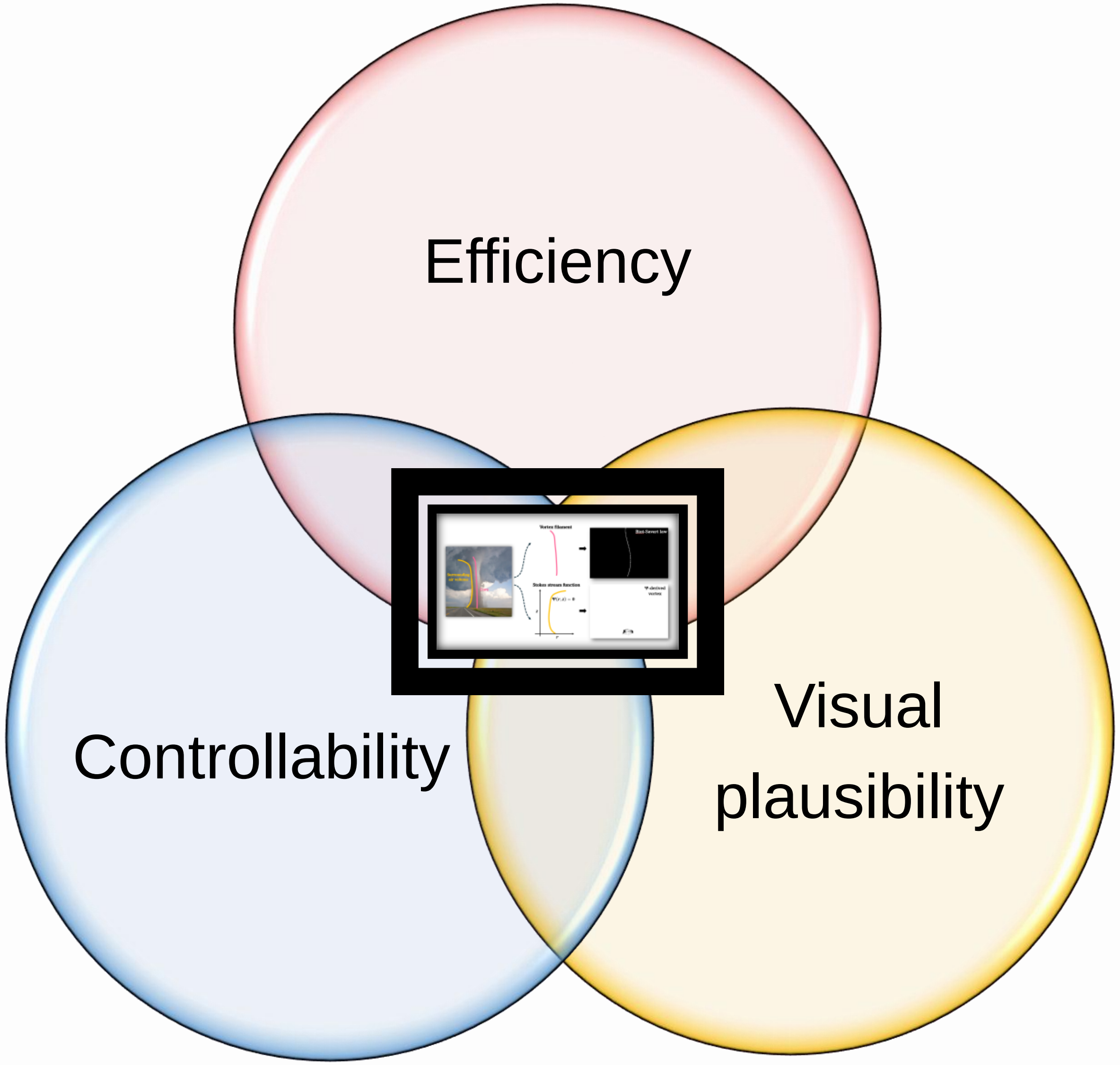


Stokes stream function





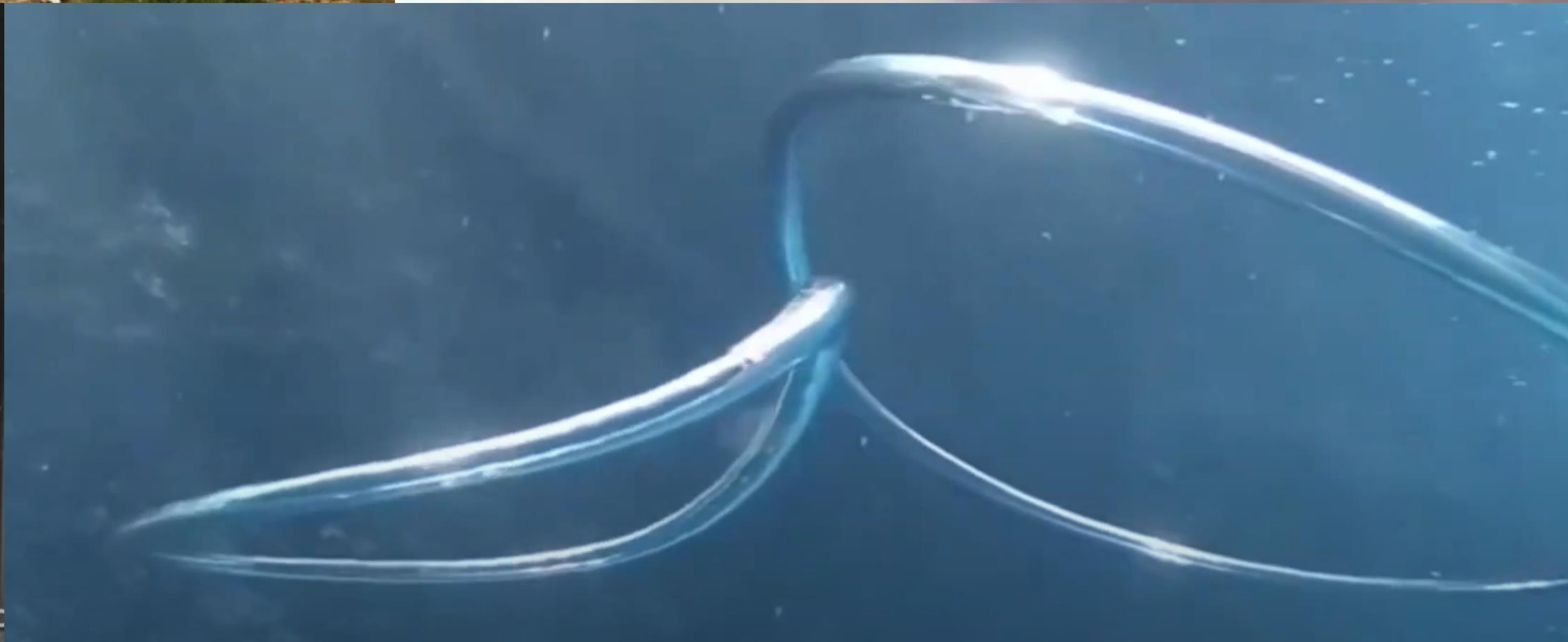
GOAL



Tornado Core

EXAMPLES OF VORTEX FILAMENT

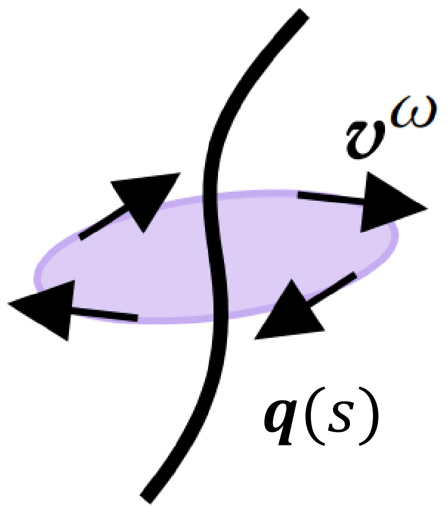
A slender, tube-like region of swirling fluid with concentrated vorticity (rotation).



VORTEX FILAMENT KINEMATICS

- Velocity around a vortex filament, i.e., Biot-Savart law

$$\boldsymbol{v}^{\omega}(\boldsymbol{p}) = -\frac{\Gamma}{4\pi}\int_0^L \frac{(\boldsymbol{p}-\boldsymbol{q}(s)) \times \partial_s \boldsymbol{q}}{(\|\boldsymbol{p}-\boldsymbol{q}(s)\|^2+\mu^2)^{\frac{3}{2}}}\,\mathrm{d}s,$$



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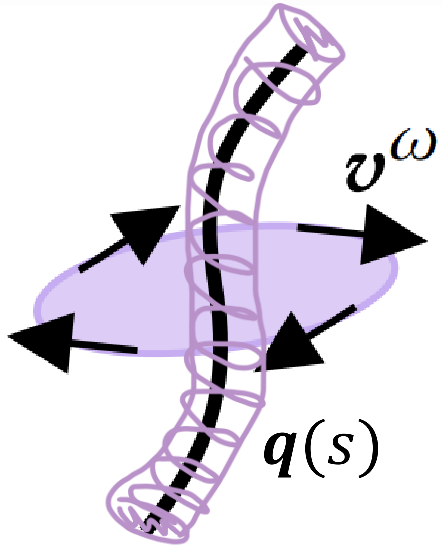

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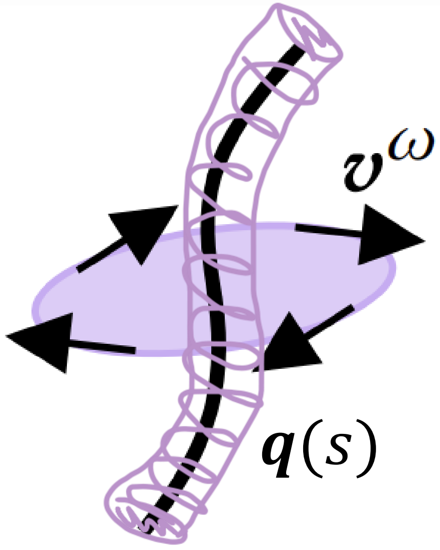
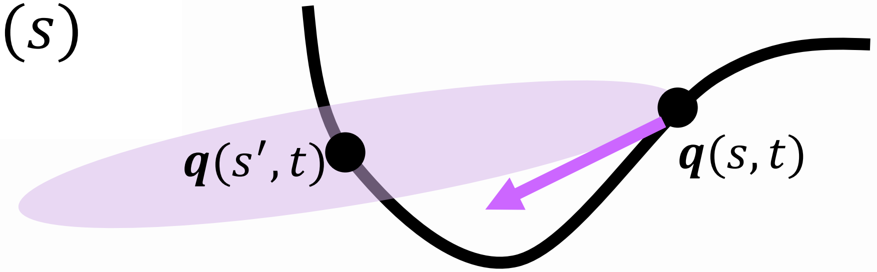
- Core kinematics by self-advection

$$\frac{\partial\boldsymbol{q}}{\partial t} = \boldsymbol{v}^{\omega}(\boldsymbol{q}) = -\frac{\Gamma}{4\pi}\int_0^L \frac{[\boldsymbol{q}(s,t)-\boldsymbol{q}(s',t)]\times\partial_{s'}\boldsymbol{q}}{(\|\boldsymbol{q}(s)-\boldsymbol{q}(s')\|)^2+\mu^2)^{\frac{3}{2}}}\mathrm{d}s'.$$





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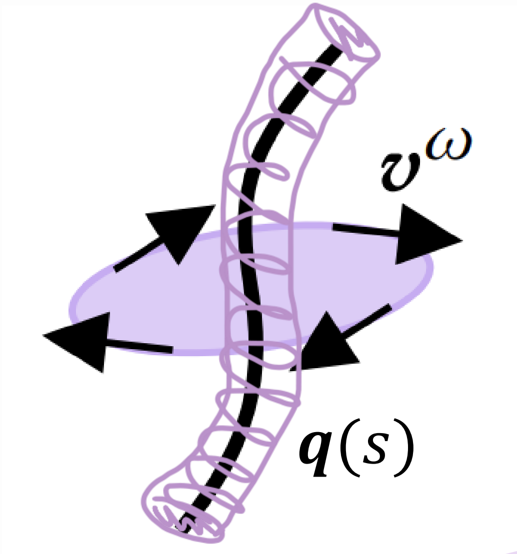

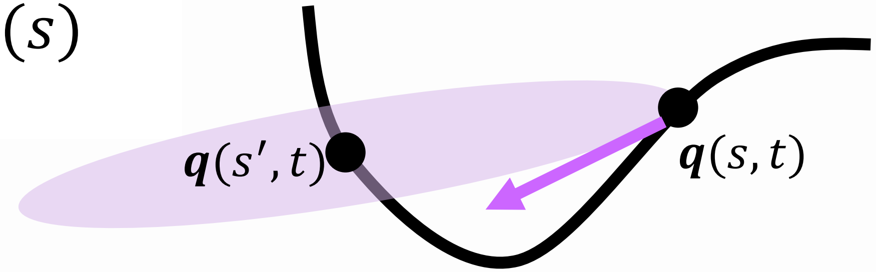

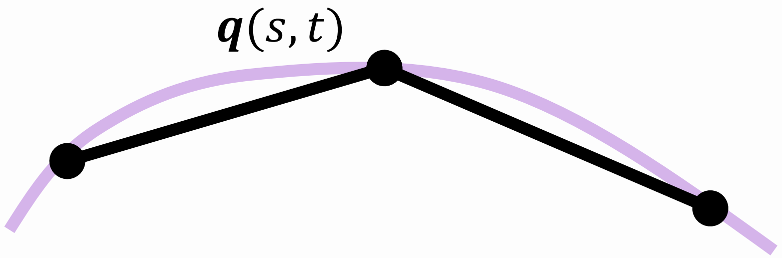

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$= 0$

- Local induction approximation [Hama 1962]

$$\frac{\partial\boldsymbol{q}}{\partial t} = \boldsymbol{v}^{LIA}(\boldsymbol{q}) + \boldsymbol{v}^{\omega}(\boldsymbol{q}) = \frac{\Gamma}{4\pi}\left(\frac{\partial\boldsymbol{q}}{\partial s}\times\frac{\partial^2\boldsymbol{q}}{\partial s^2}\right)\log\left(\frac{2\sqrt{l_- l_+}}{e^{1/4}a}\right)$$
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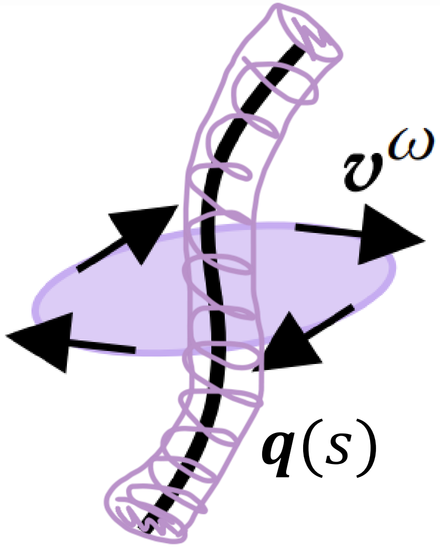

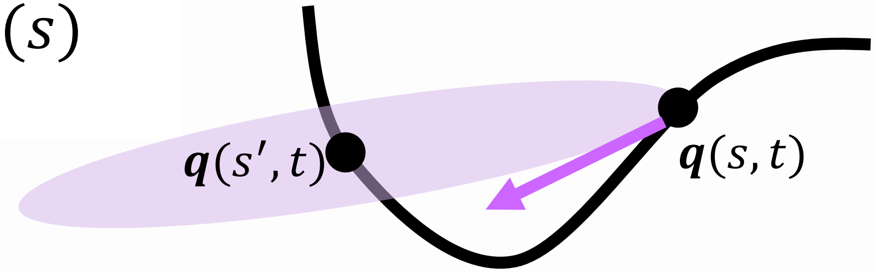

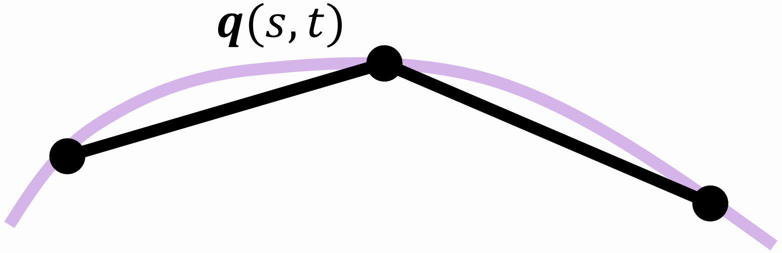

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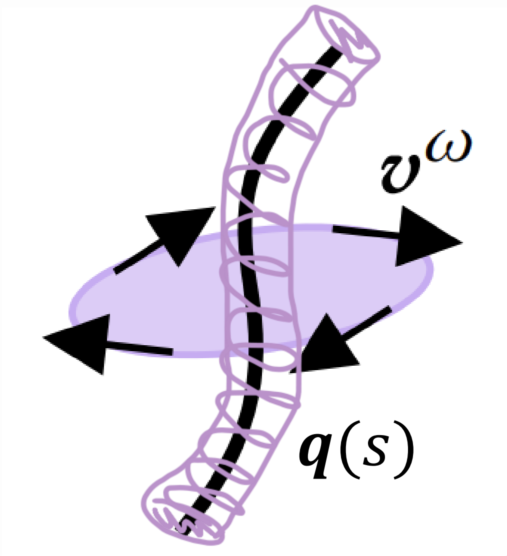
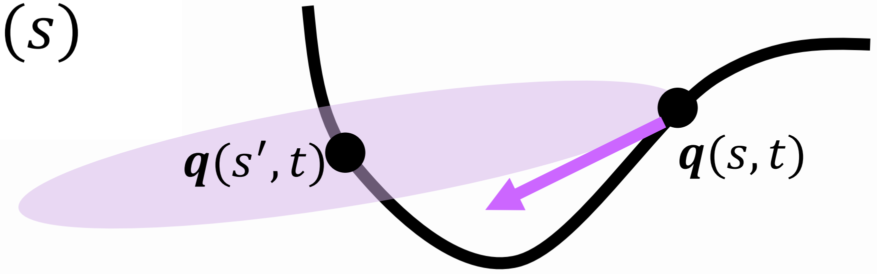
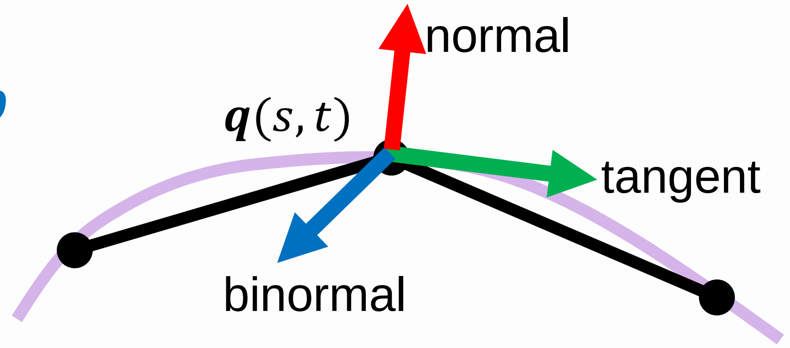
$$-\frac{\Gamma}{4\pi}\int_0^L \frac{(\boldsymbol{q}(s)-\boldsymbol{q}(s')) \times \partial_{s'}\boldsymbol{q}}{(\|\boldsymbol{q}(s)-\boldsymbol{q}(s')\|^2 + \mu^2)^{\frac{3}{2}}}\,\mathrm{d}s',$$







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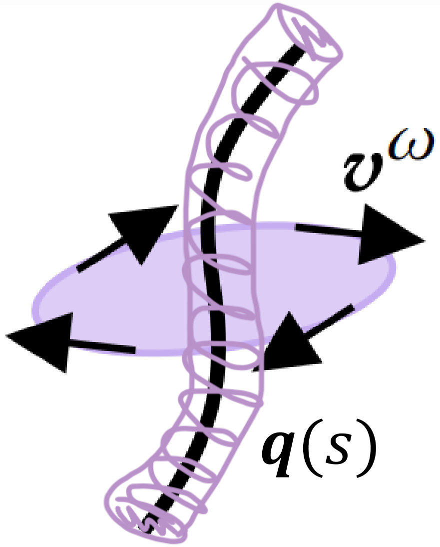
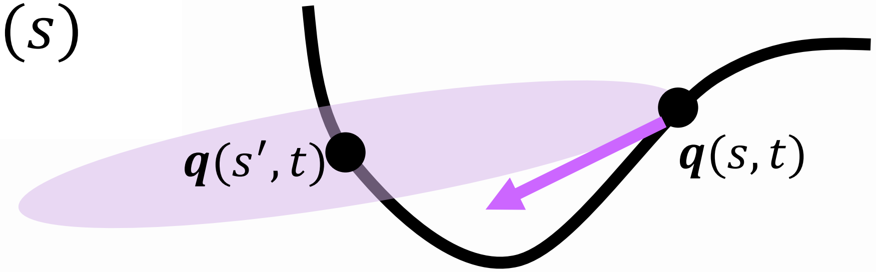
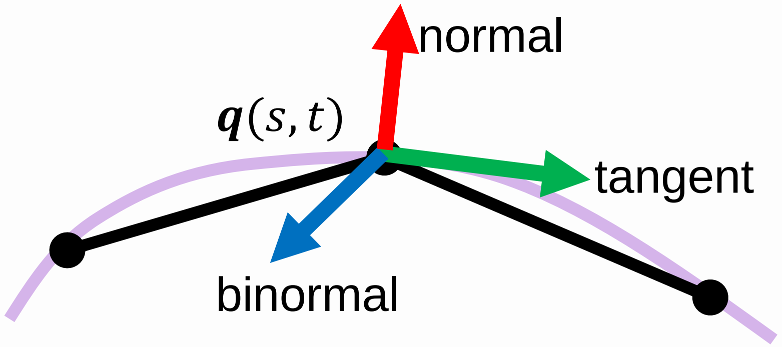
$$-\frac{\Gamma}{4\pi}\int_0^L \frac{(\boldsymbol{q}(s)-\boldsymbol{q}(s'))\times\partial_{s'}\boldsymbol{q}}{(\|\boldsymbol{q}(s)-\boldsymbol{q}(s')\|^2+\mu^2)^{\frac{3}{2}}}\,\mathrm{d}s',$$

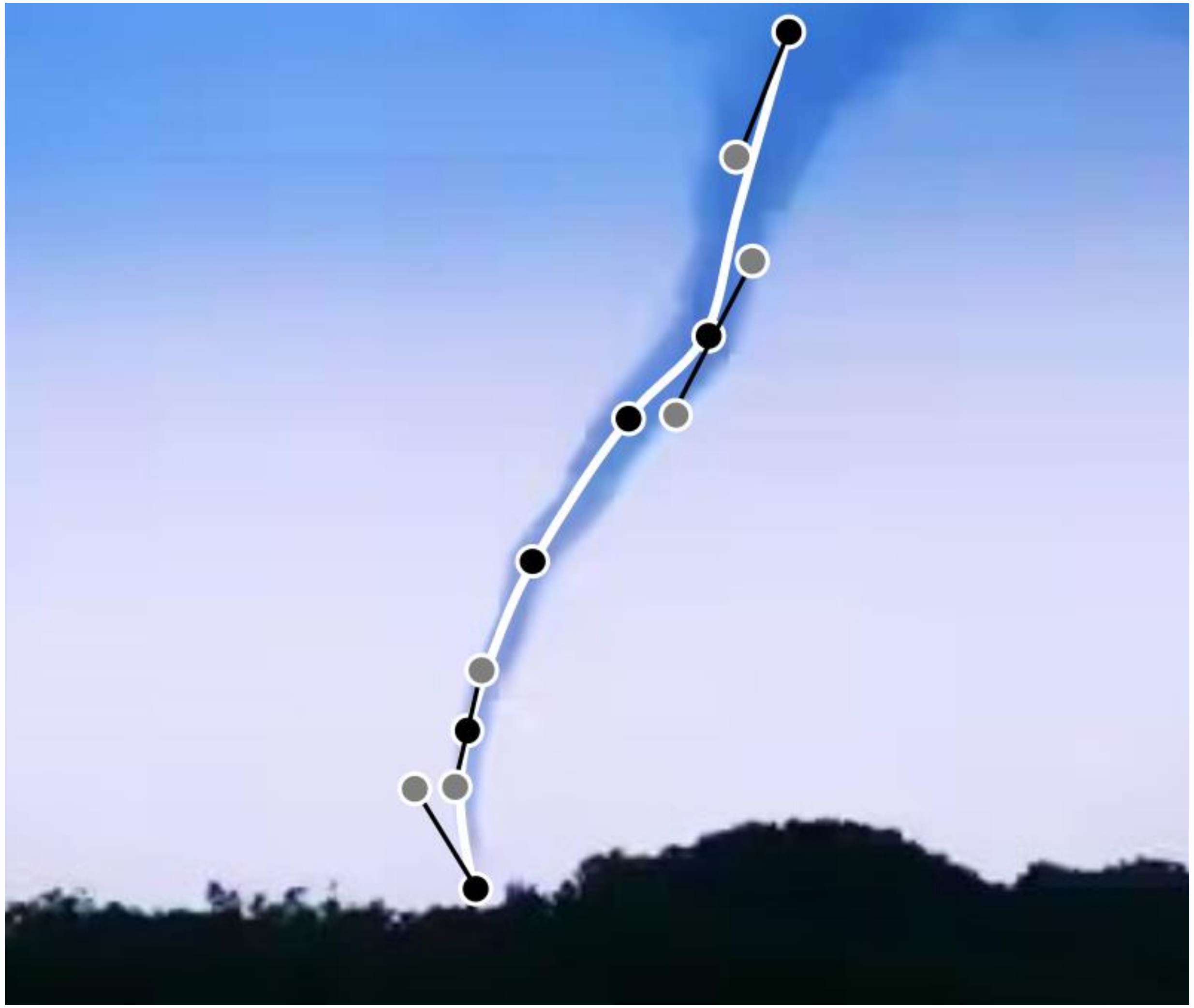
- Global environment velocity



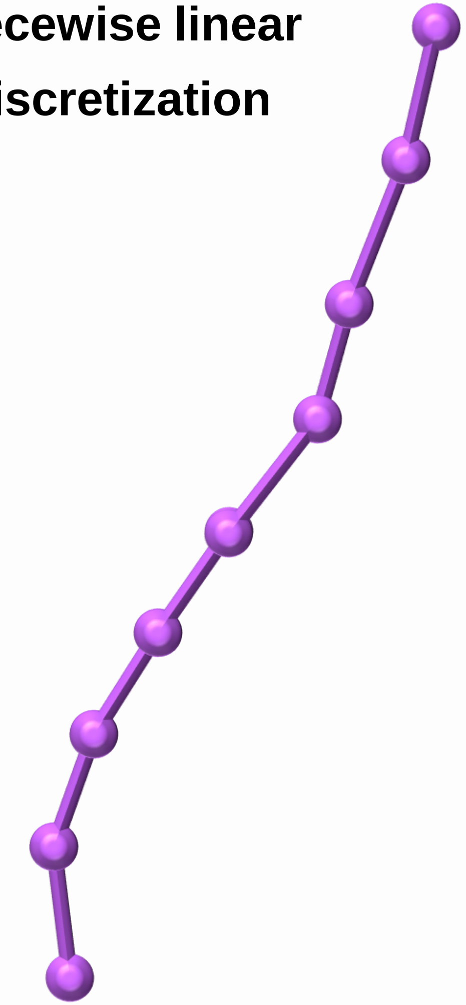




INITIALIZATION AND DISCRETIZATION



Piecewise linear discretization



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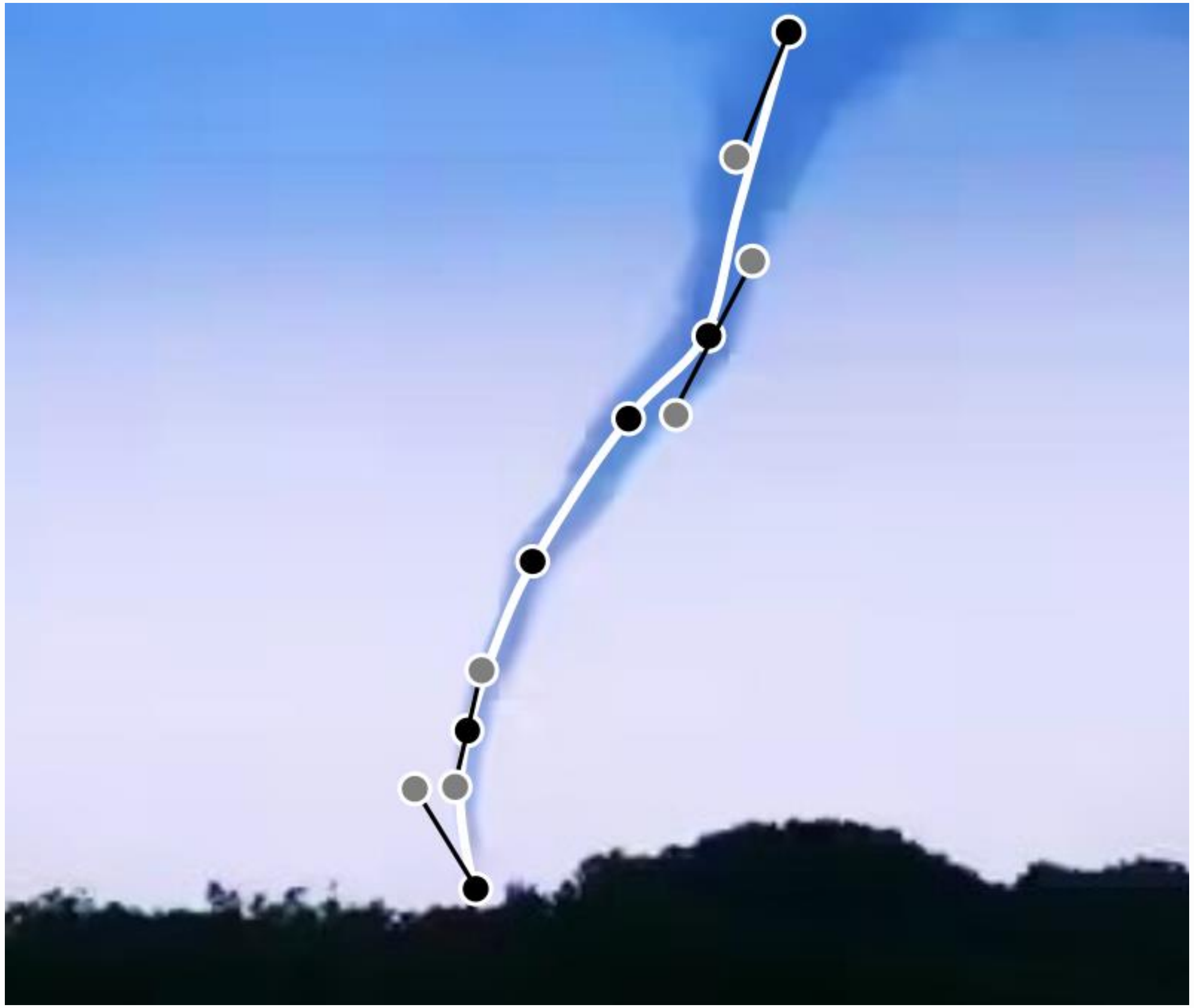

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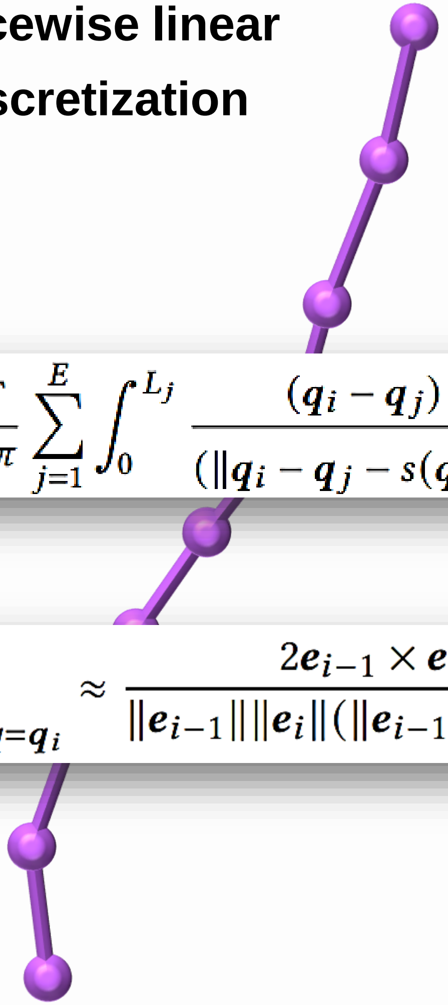


Velocity

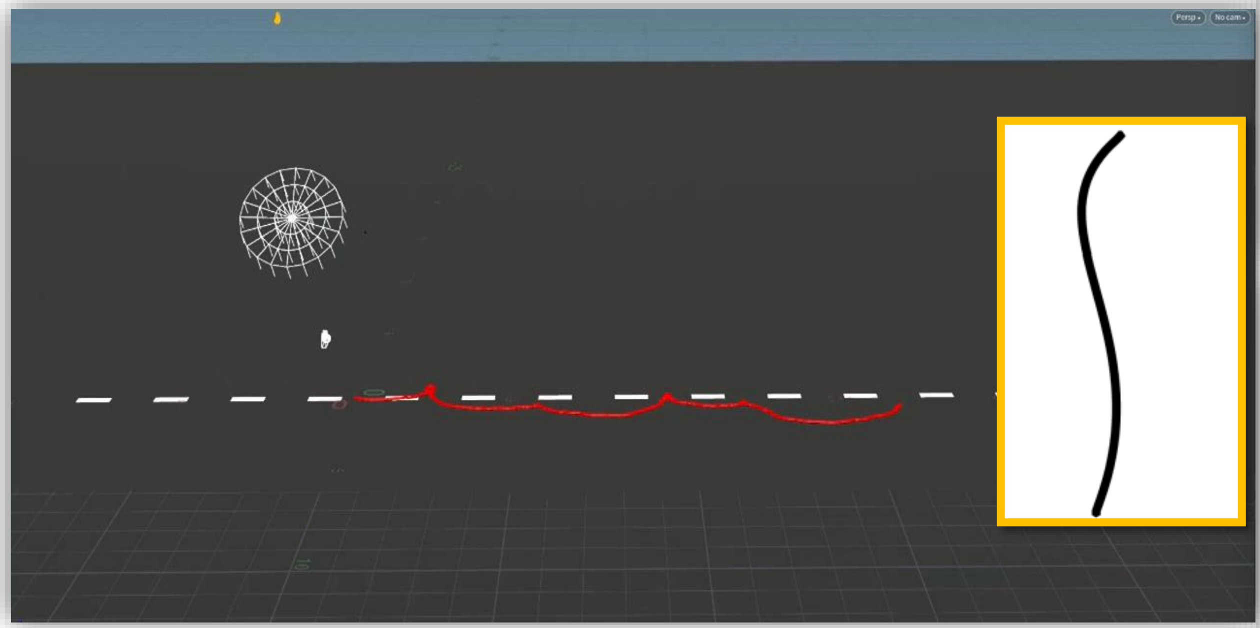
$$v^{\omega}(q_i) = -\frac{\Gamma}{4\pi}\sum_{j=1}^{E}\int_0^{L_j}\frac{(q_i - q_j)\times(q_{j+1}-q_j)/L_j}{(\|q_i - q_j - s(q_{j+1}-q_j)/L_j\|^2+\mu^2)^{\frac{3}{2}}}\mathrm{d}s,$$

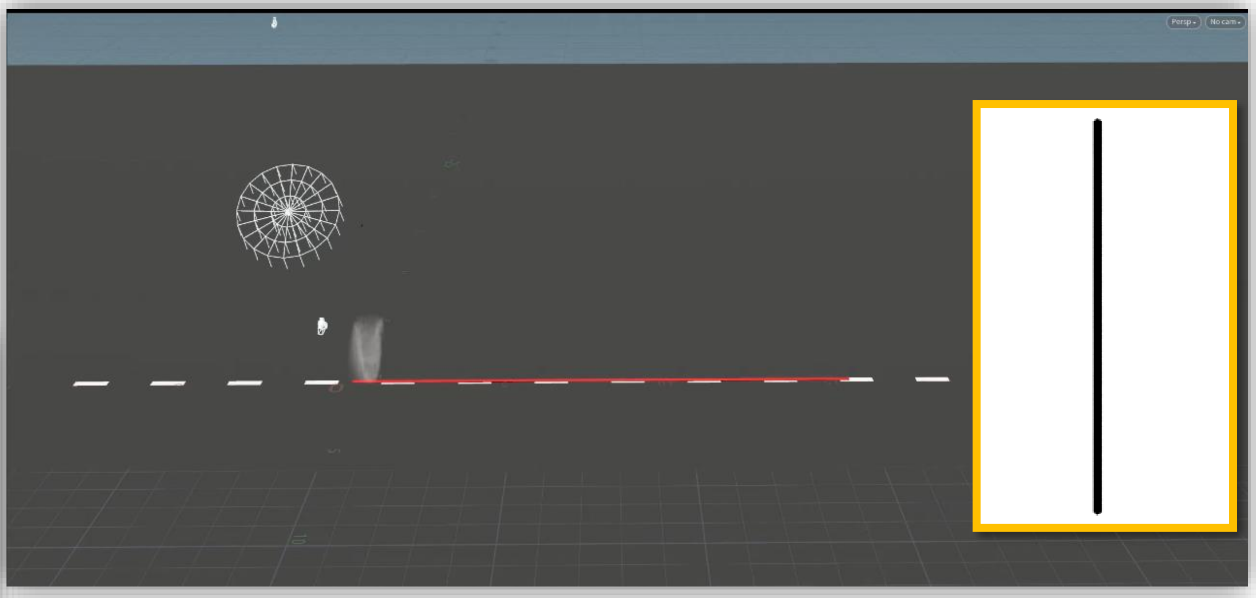
LIA

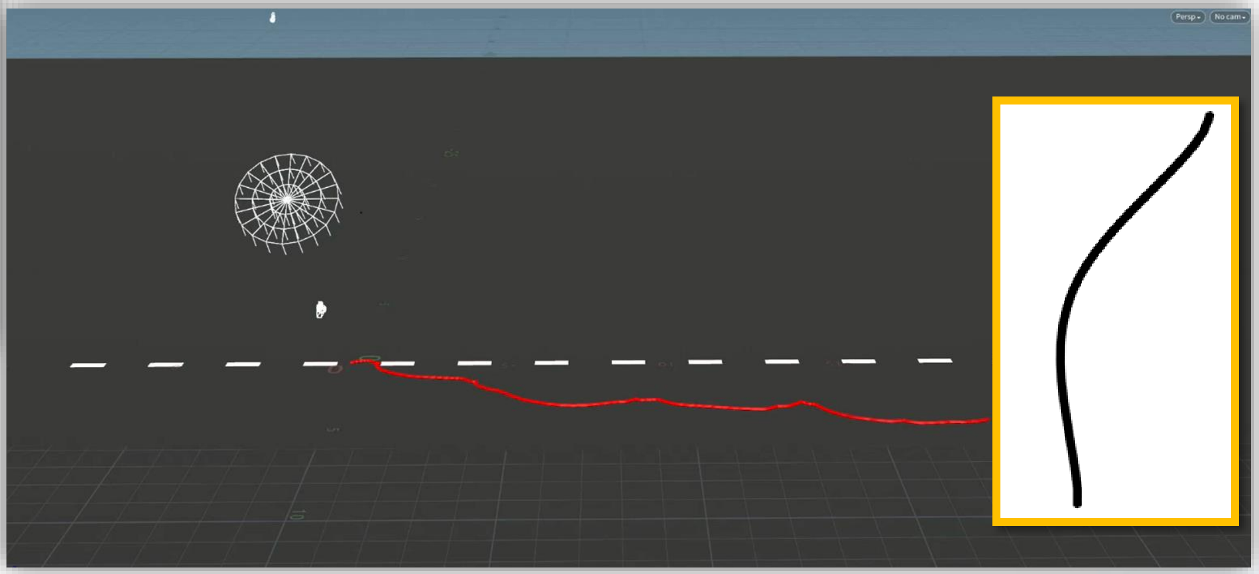
$$\left.\frac{\partial q}{\partial s}\times\frac{\partial^2 q}{\partial s^2}\right|_{q=q_i}\approx\frac{2e_{i-1}\times e_i}{\|e_{i-1}\|\|e_i\|(\|e_{i-1}\|+\|e_i\|)},\quad e_i = q_{i+1}-q_i.$$

KINEMATICS FROM DIFFERENT INITIALIZATION

Curvature decides everything!

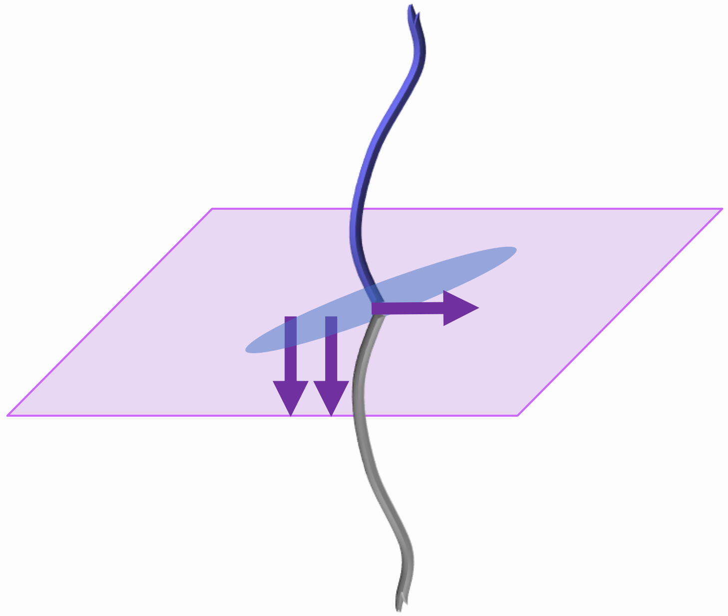


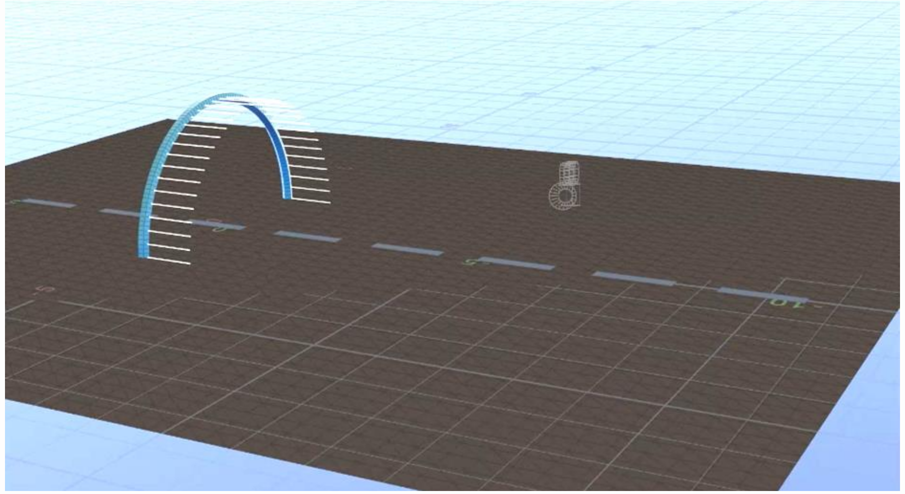




BOUNDARY CONDITION

- Boundary layer effect
  - Friction forces (viscosity, surface roughness) and centrifugal forces
  - High resolution discretization
- Non-penetration condition $\boldsymbol{n}^t \dot{\boldsymbol{q}}(s)|_{s=0} = 0$
  - For fundamental mass conservation law
  - Mirroring the filament [Schwarz 1985]





Water condensation funnel

AXISYMMETRIC FLOW

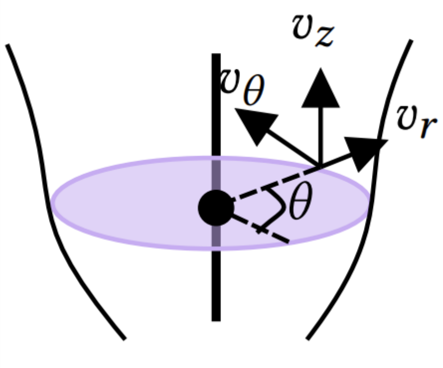
- Describe axisymmetric in cylindrical coordinates
  - Augment swirl flow (v_θ) with **radial** (v_r) and **axial** flow (v_z)
- Analytical vortex solution derived from the NS equations
  - Rankine vortex $v_r = 0, \quad v_\theta(r) = \frac{\Gamma}{2\pi}\begin{cases} r/a^2 & r \le a, \\ 1/r & r > a \end{cases}, \quad v_z = 0$
  - Burgers vortex $v_r = -\alpha r, \quad v_z = 2\alpha z, \quad v_\theta = \frac{\Gamma}{2\pi r} g(r)$
  - Shtern's solution [Shtern et al. 1997]

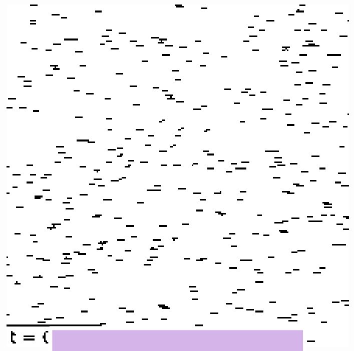
$$\begin{cases} v_r = \mu \,\mathrm{Re}/r, \\ v_\theta = \mu\Gamma/r, \\ v_z = \mu\left[W_c + W_p r^2 + W_r r^{\mathrm{Re}}\right]. \end{cases}$$

Re: Reynolds number
Γ: Circulation
μ: Kinematic viscosity
W_c, W_p, W_r: shear constants

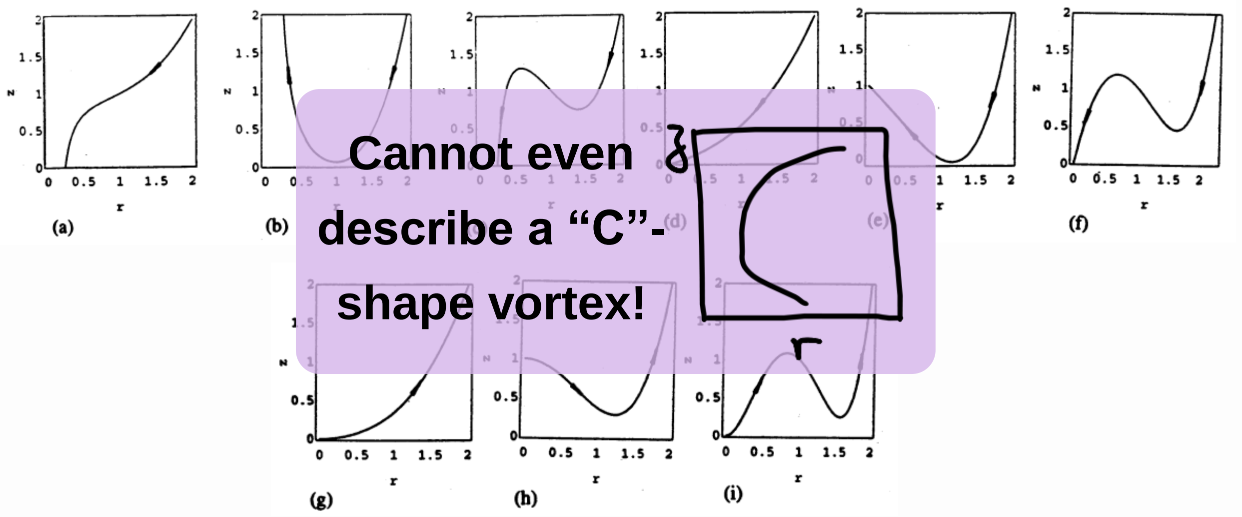
  - Funnel shape: zero level set of the **Stokes stream function**

$$\Psi(r,z) = \frac{W_c\mu}{2} r^2 + \frac{W_p\mu}{4} r^4 + \frac{W_r\mu}{2+\mathrm{Re}} r^{\mathrm{Re}+2} - \mathrm{Re}\,\mu z = 0.$$





Rankine vortex



SPLINE-BASED VORTEX SOLUTION

- Let user design the Stokes stream function!

Radial and axial velocity

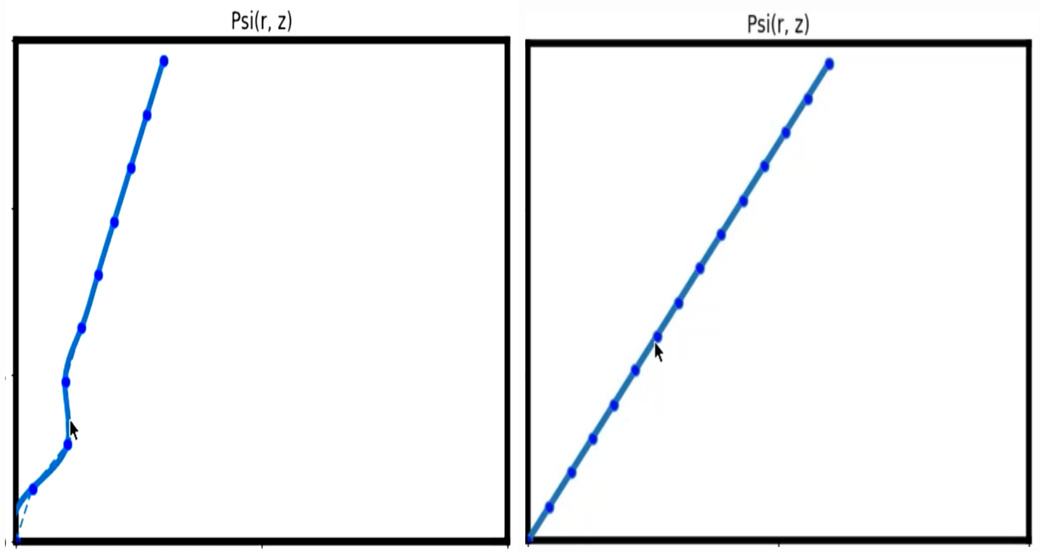
$$\begin{cases} v_r = -\frac{1}{r}\frac{\partial \Psi}{\partial z}, \\ v_z = +\frac{1}{r}\frac{\partial \Psi}{\partial r}. \end{cases}$$

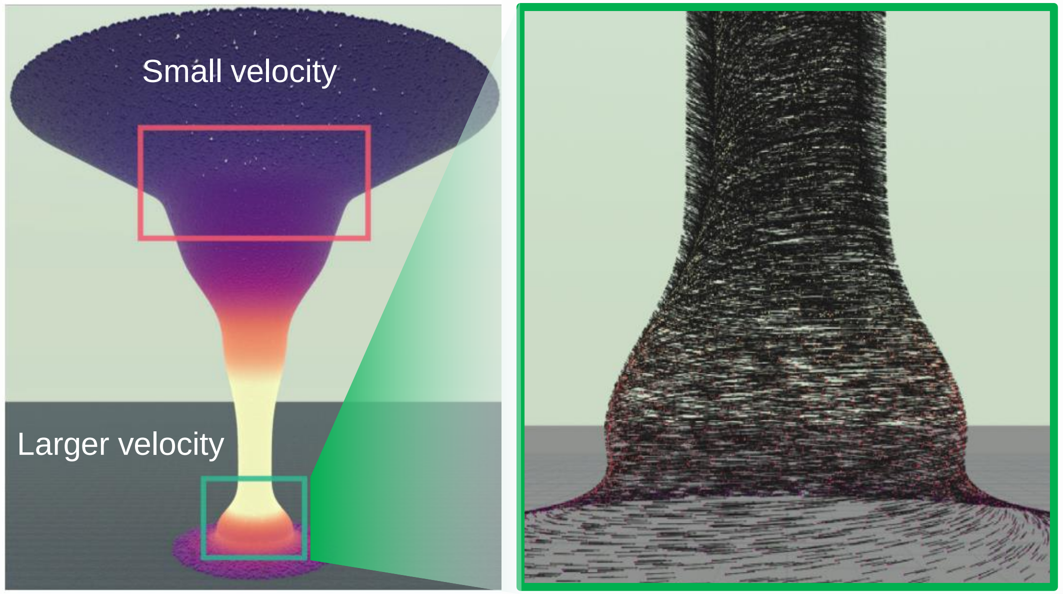
Continuity equation

$$\frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{\partial v_z}{\partial z} = 0,$$

- No swirl velocity v_θ
- Couple with swirl velocity induced by the core

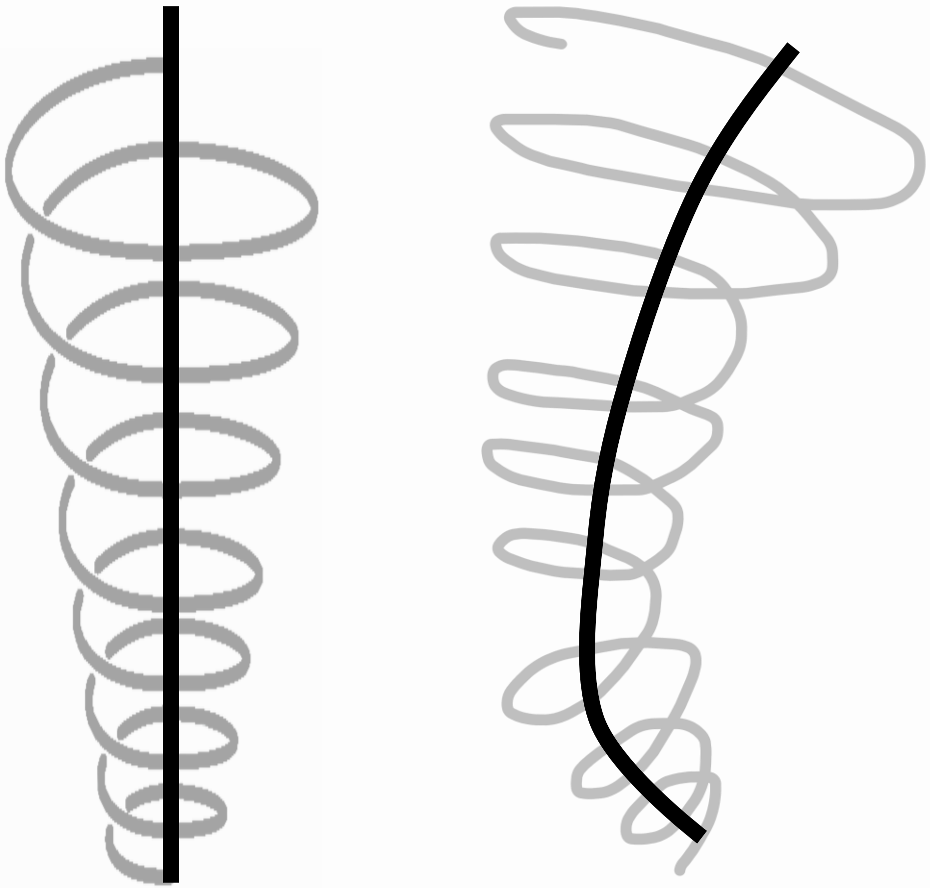
$$\begin{cases} v_r = \frac{\alpha}{r}\nabla_z f, \\ v_\theta = \frac{\Gamma}{2\pi r}, \\ v_z = \frac{\alpha}{r}. \end{cases}$$





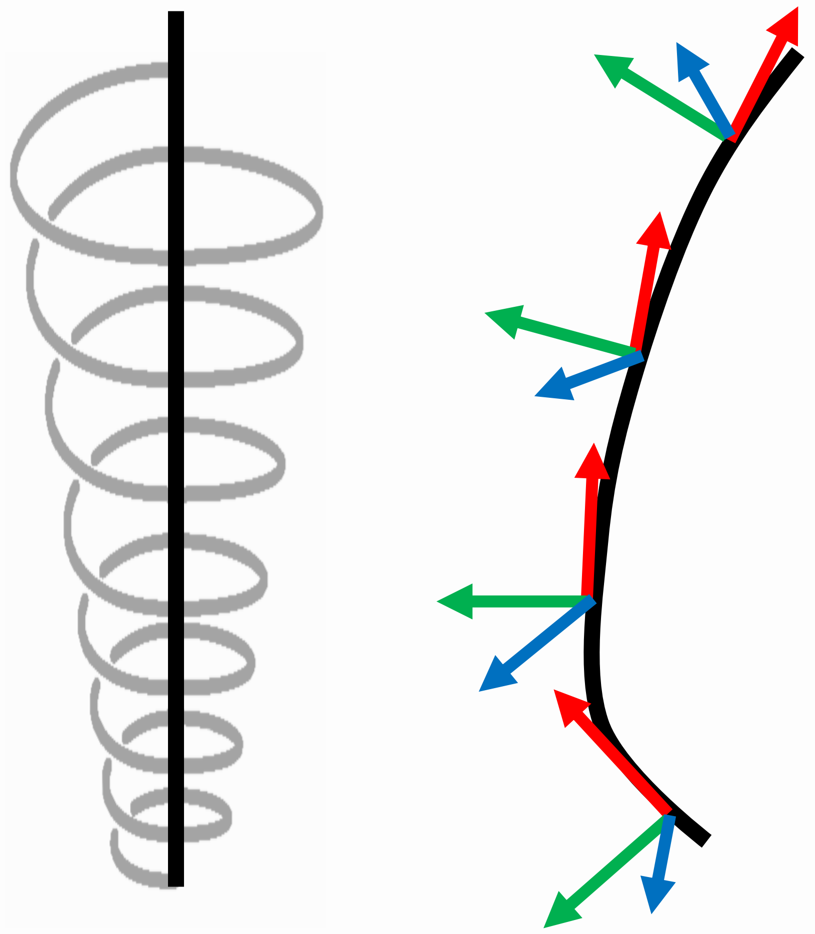
ADAPTING VORTEX TO A CURVED CORE

- From a rectilinear core to a curved one



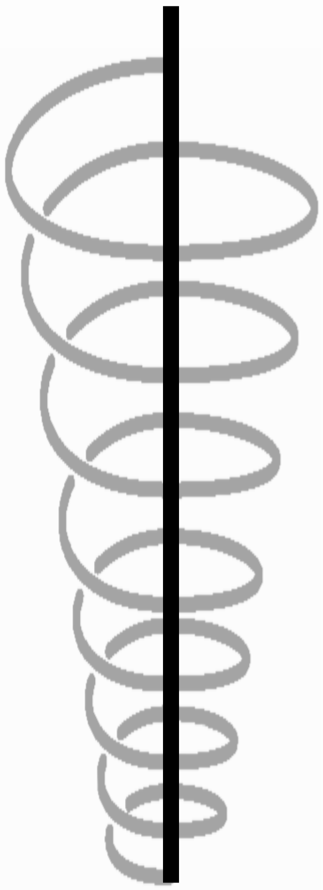
ADAPTING VORTEX TO A CURVED CORE

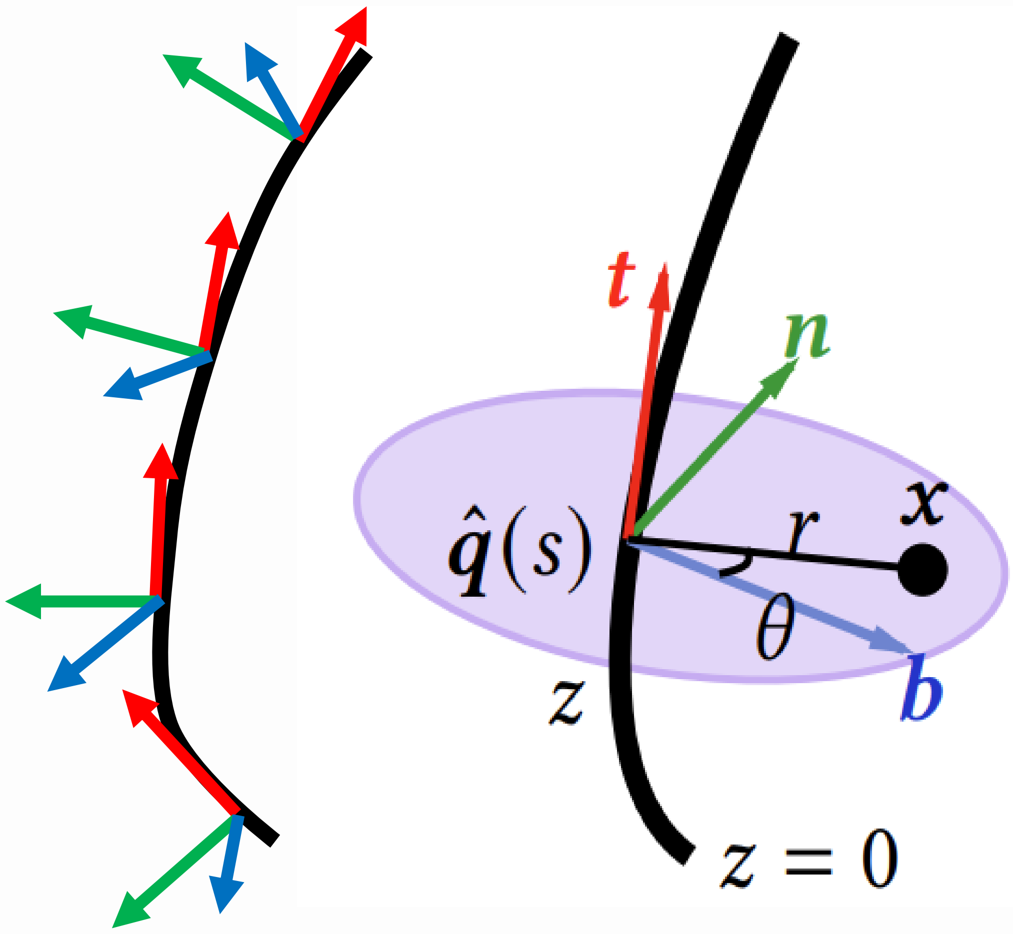
- From a rectilinear core to a curved one
  - Update the material frame



ADAPTING VORTEX TO A CURVED CORE

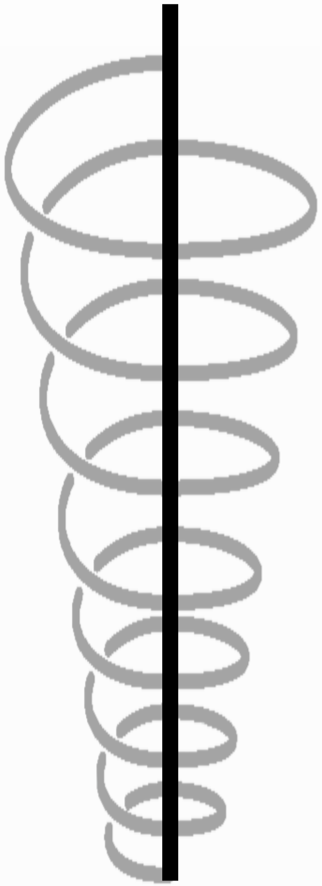
- From a rectilinear core to a curved one
  - Update the material frame
  - Compute local cylindrical coordinates at the given point

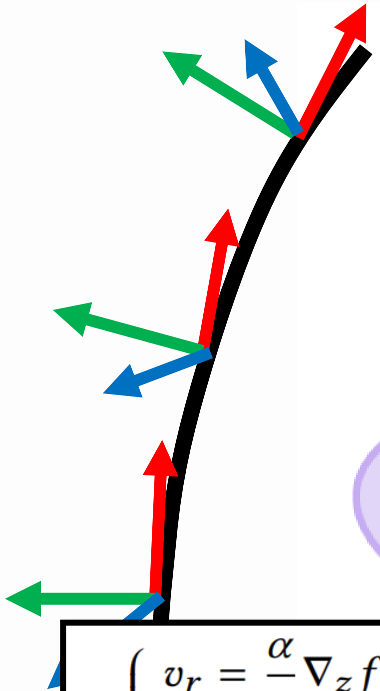




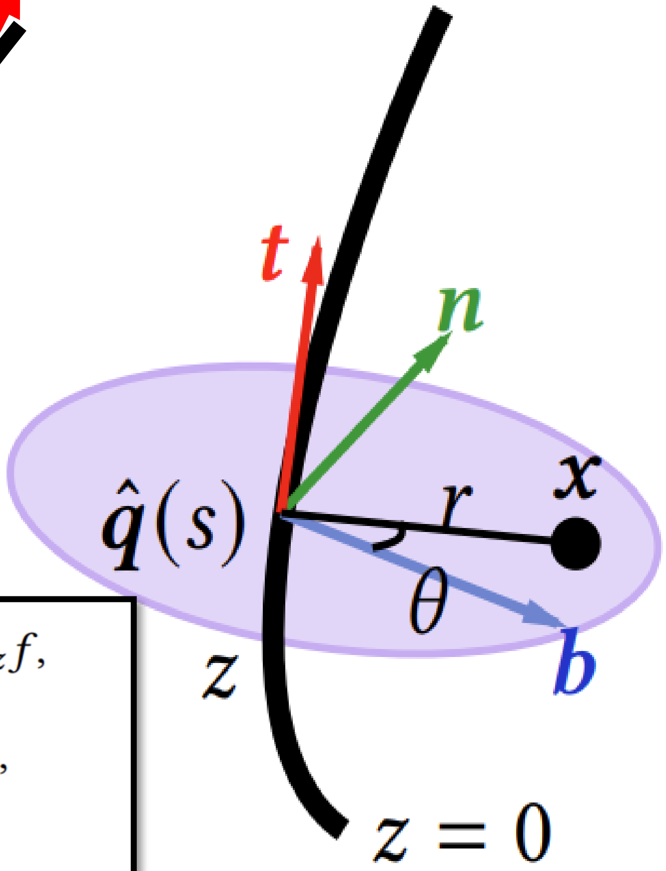
ADAPTING VORTEX TO A CURVED CORE

- From a rectilinear core to a curved one
  - Update the material frame
  - Compute local cylindrical coordinates at the given point
  - Compute the vortex velocity in the local frame





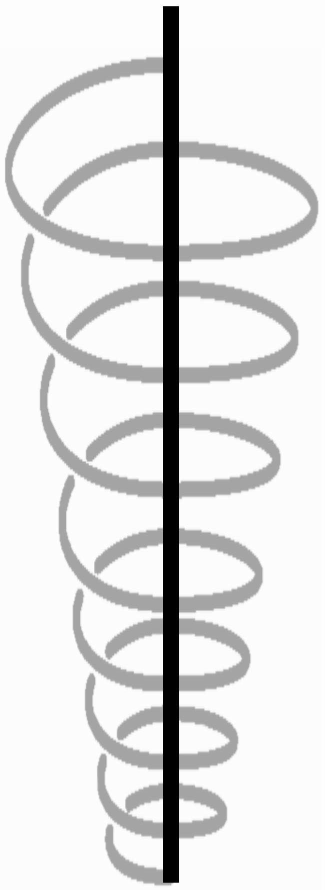
$$\begin{cases} v_r = \dfrac{\alpha}{r}\nabla_z f, \\ v_\theta = \dfrac{\Gamma}{2\pi r}, \\ v_z = \dfrac{\alpha}{r}. \end{cases}$$

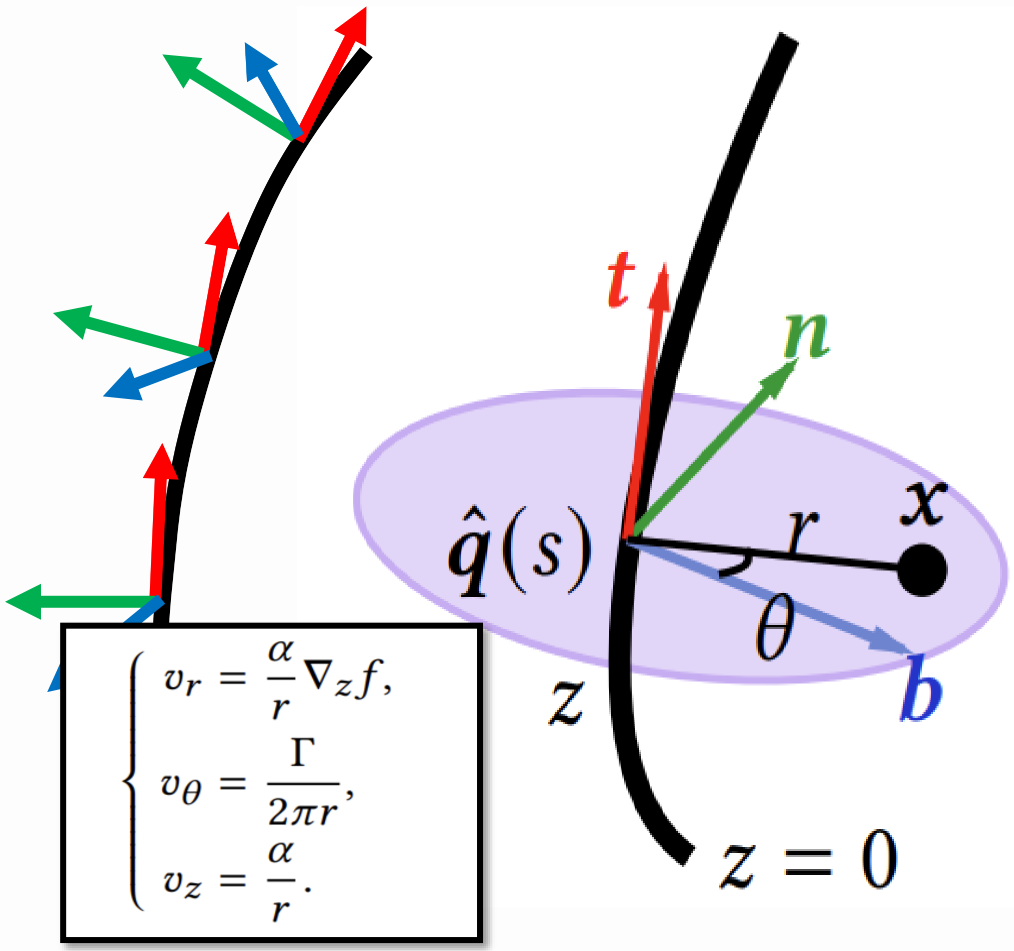


ADAPTING VORTEX TO A CURVED CORE

- From a rectilinear core to a curved one
  - Update the material frame
  - Compute local cylindrical coordinates at the given point
  - Compute the vortex velocity in the local frame
  - Transform the local velocity to the global frame

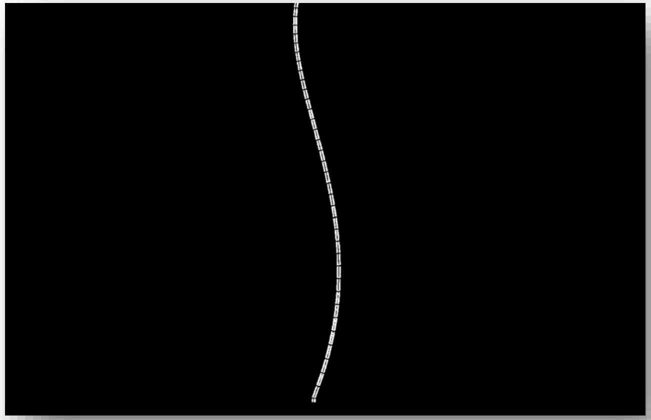
$$\boldsymbol{v}(\boldsymbol{x}) = \big(\boldsymbol{b}(s)\ \boldsymbol{n}(s)\ \boldsymbol{t}(s)\big) \begin{pmatrix} \cos(\theta)v_r - \sin(\theta)v_\theta \\ \sin(\theta)v_r + \cos(\theta)v_\theta \\ v_z \end{pmatrix}.$$

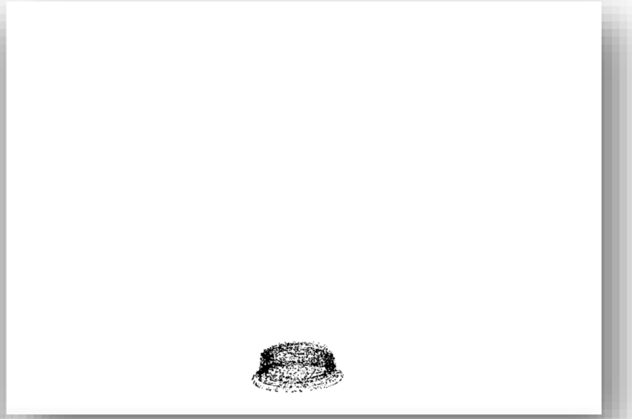


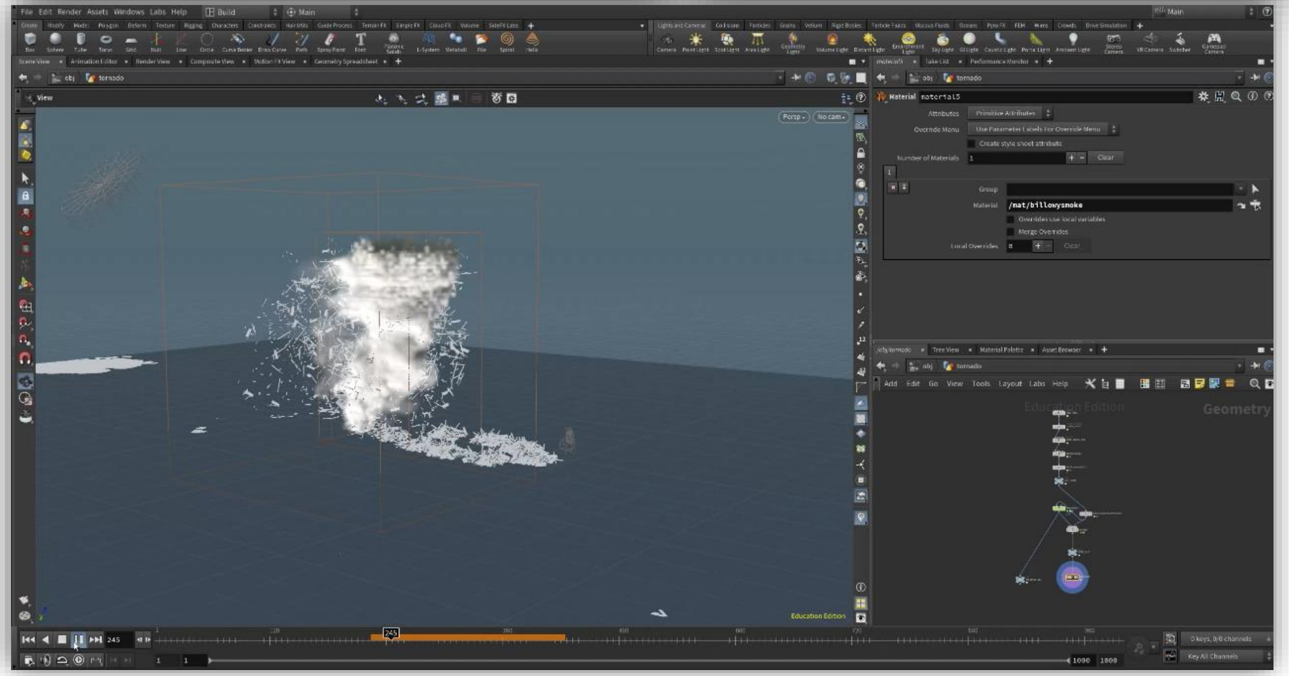


IMPLEMENTATION

- Numerical integration (C++)
  - Compute the kinematics of the core
  - Randomly sample particles around the core, and advect them in the vortex velocity field
  - Less than 30ms per step for 15k particles
- Procedural refinement (Houdini)
  - Import the particles and their attributes
  - Add turbulence
  - Transport rigid bodies
  - Volume rendering
- Example code
  - https://gitlab.inria.fr/geomerix/public/twisterforge

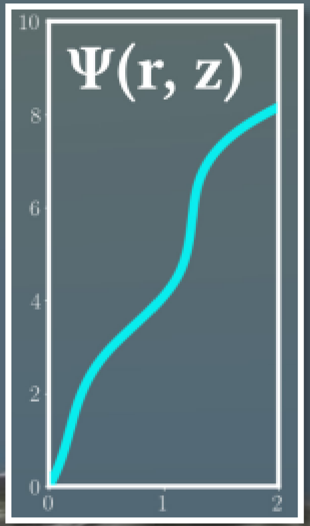


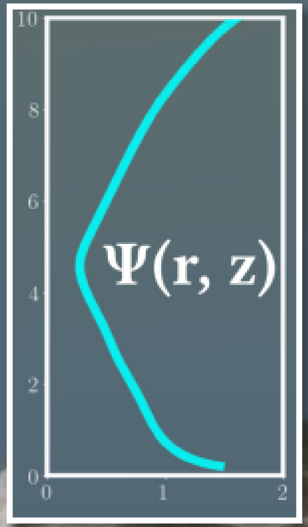





NON-EVEN TERRAIN
Ψ(r, z)
10
8
6
4
2
0
0.0
0.5
1.0
1.5
2.0

CONTROL OVER THE FUNNEL PROFILE

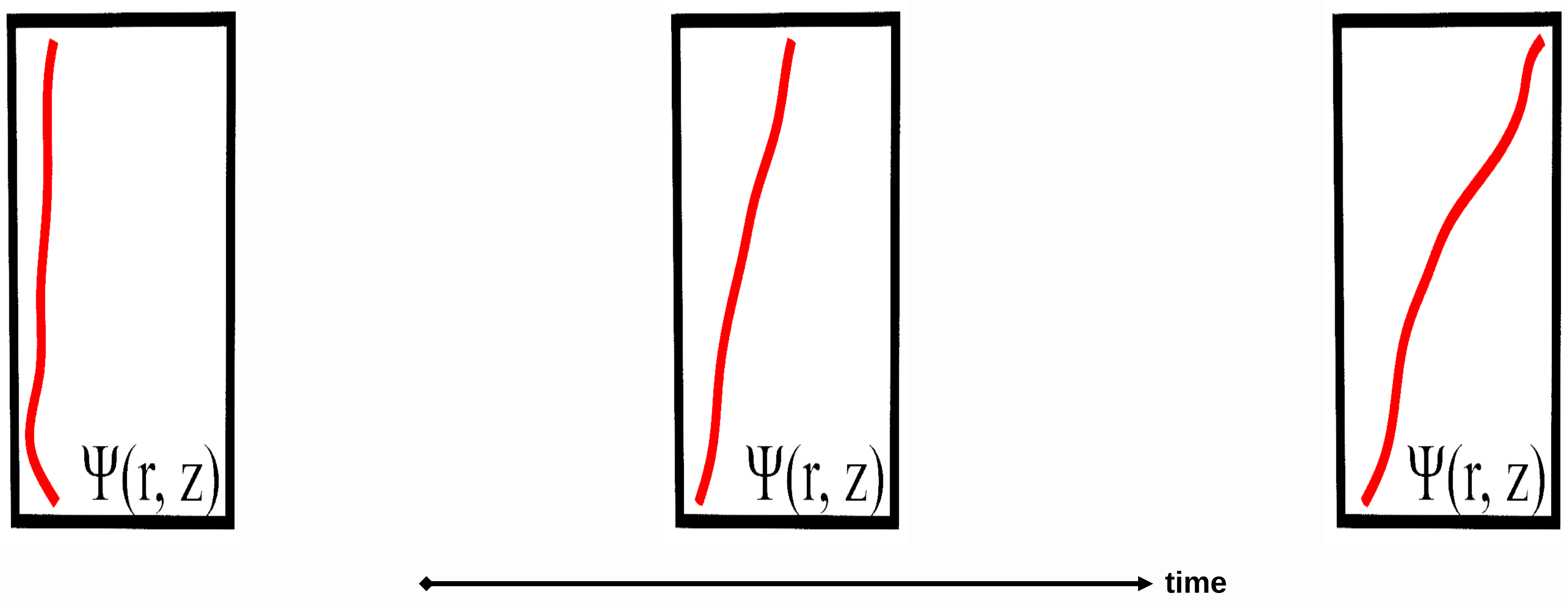




CONTROL OVER THE FUNNEL PROFILE

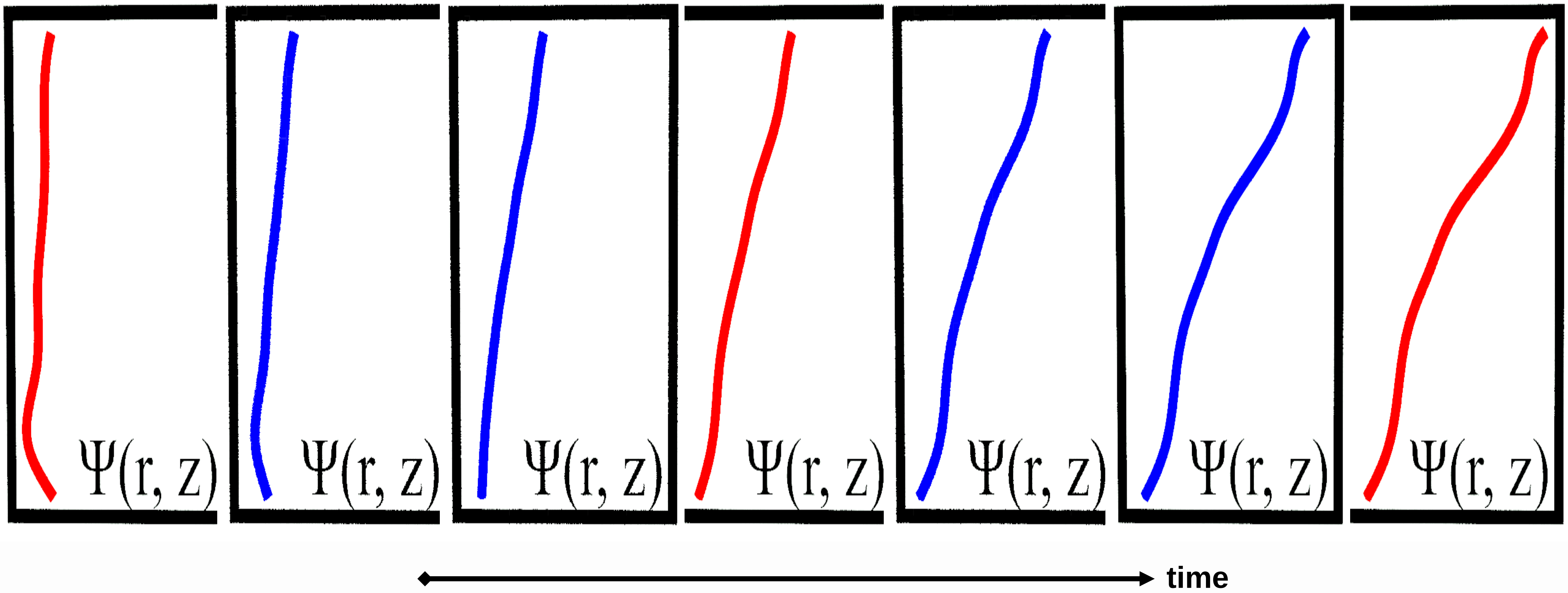
FROM STEADY TO NON-STEADY FLOW

- Spline parameterization allows for easy interpolation in time



FROM STEADY TO NON-STEADY FLOW

- Spline parameterization allows for easy interpolation in time

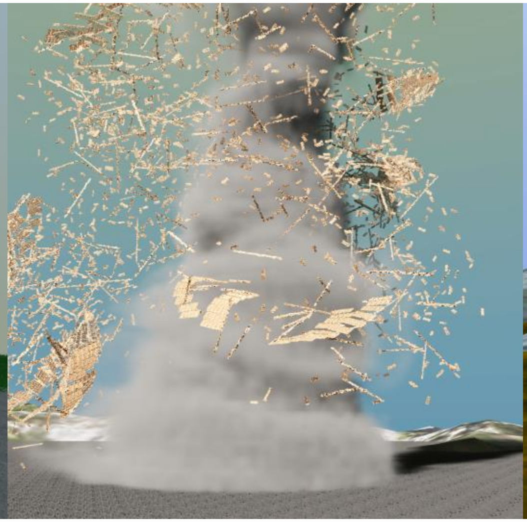
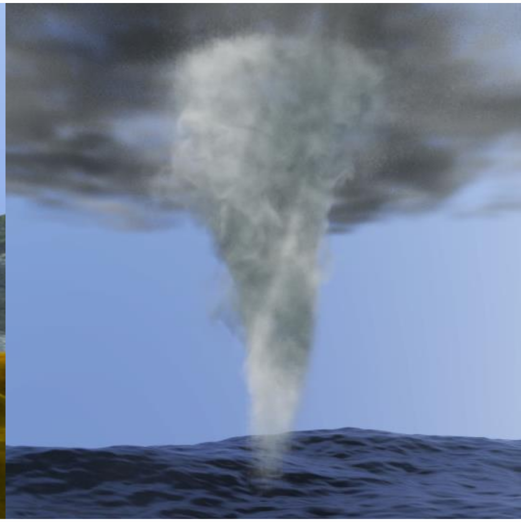


INTERPOLATING THE PROFILE

IMITATING THE LIFE CYCLE

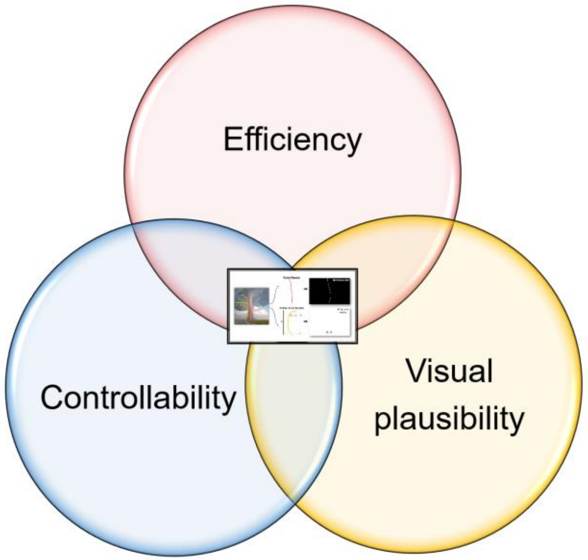
VIRTUAL VS. REAL

CONCLUSION

- Contribution
  - A flexible authoring tool for controllable, efficient and plausible tornado animations
- Limitations
  - Our vortex model is not physically accurate
    - The spline-derived vortex violates the NS equation in general
    - Temporal interpolation can be far from the real dynamics
    - The core and the funnel are loosely coupled
  - Boundary conditions are oversimplified
    - More to consider, such as viscosity, friction, surface roughness
  - Interaction with solid bodies
    - Two-way coupling, i.e., solids should affect the tornado dynamics as well



Thanks!

MIG 2024 Arlington