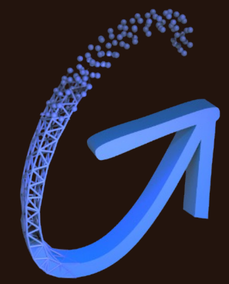
SIGGRAPH 2023
LOS ANGELES+ 6-10 AUG

THE PREMIER CONFERENCE & EXHIBITION ON
COMPUTER GRAPHICS & INTERACTIVE TECHNIQUES

SOMIGLIANA COORDINATES

AN ELASTICITY-DERIVED APPROACH FOR CAGE DEFORMATION



JIONG CHEN, ECOLE POLYTECHNIQUE
FERNANDO DE GOES, PIXAR ANIMATION STUDIOS
MATHIEU DESBRUN, INRIA

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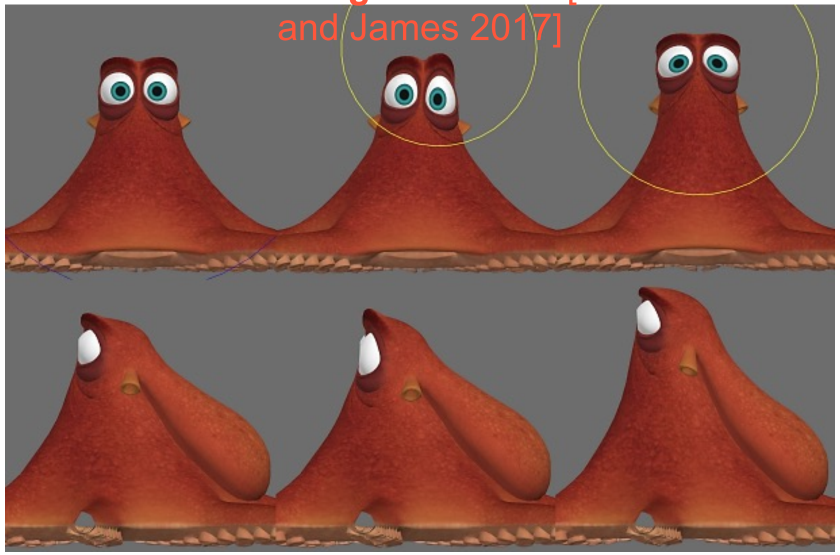
REAL-TIME MESH DEFORMATION

- Sculpting brushes using fundamental solutions of elasticity
  - based on regularized Kelvinlets

Multiscale regularization [de Goes and James 2017]



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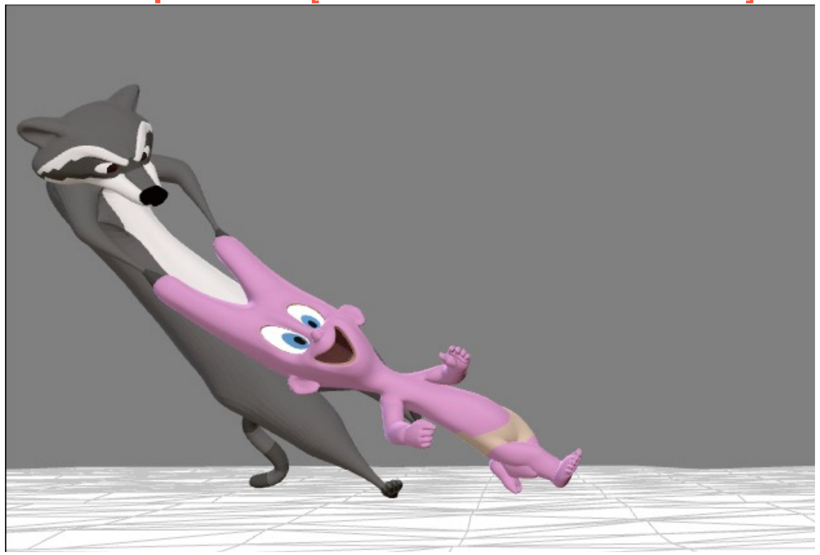

Anisotropic and multi-frequency regularization [Chen and Desbrun 2022]

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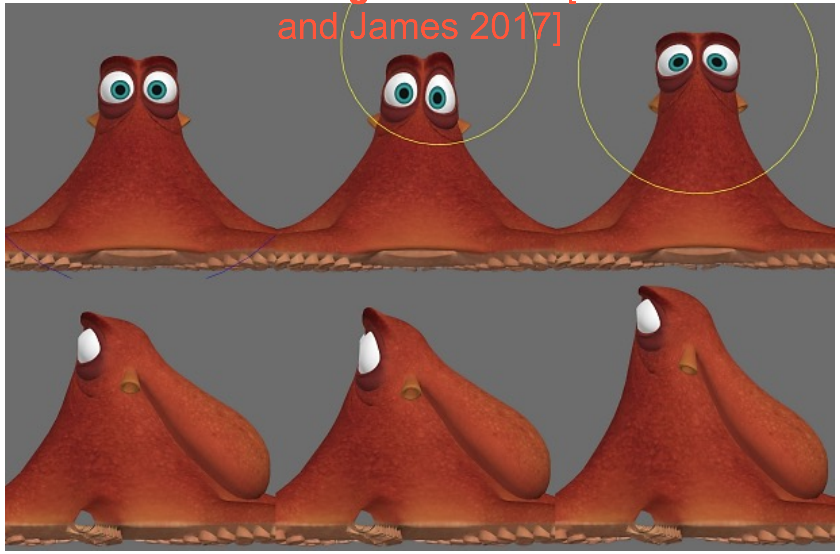
  - **Meshfree, control of volume change, and extremely fast**



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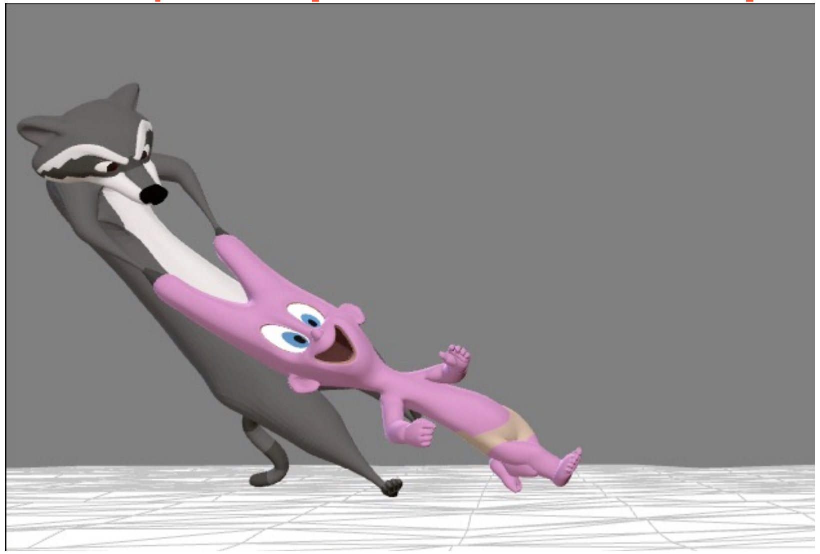

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  - **Unaware of boundaries!**

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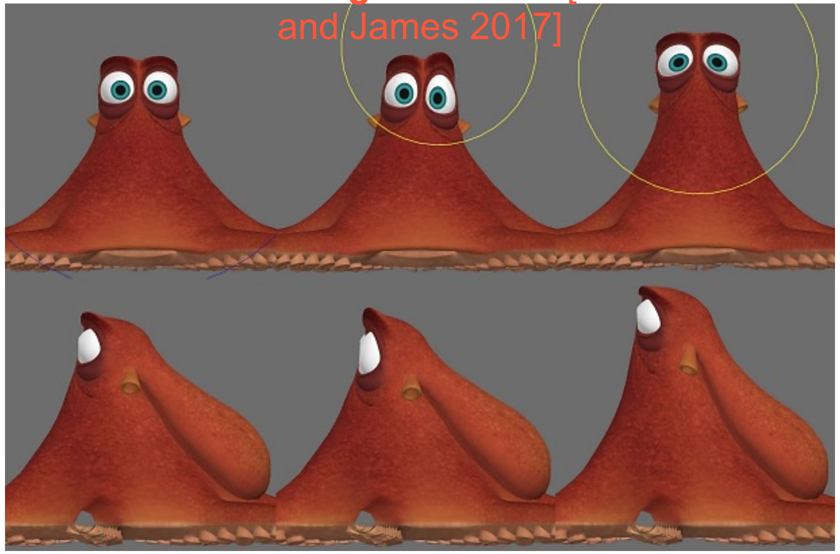

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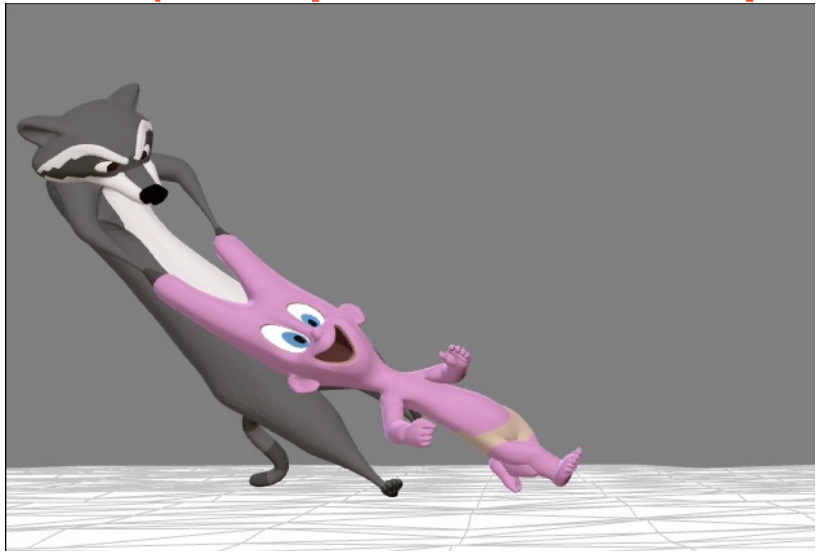

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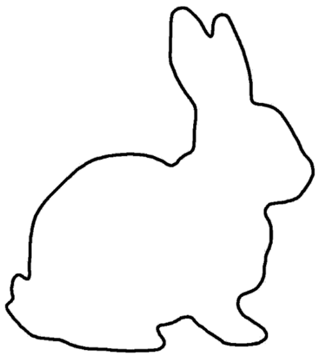

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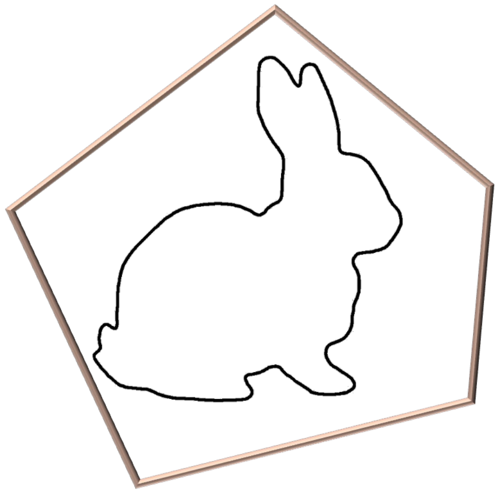

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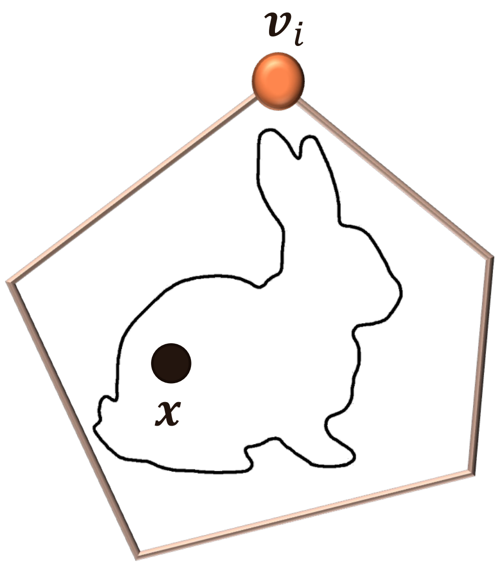

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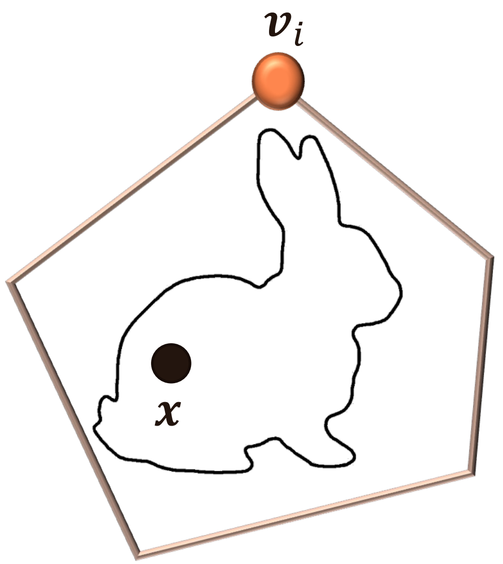

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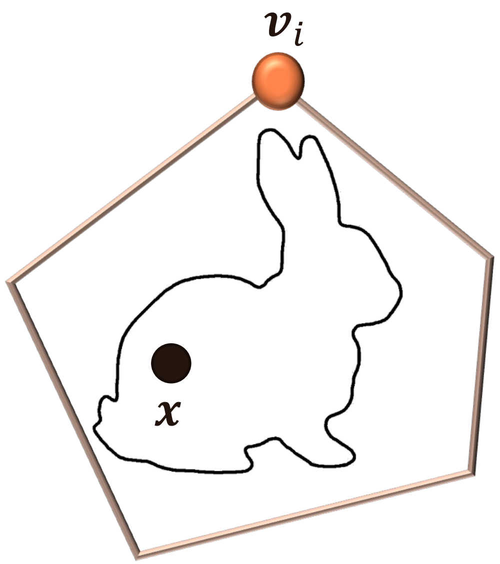

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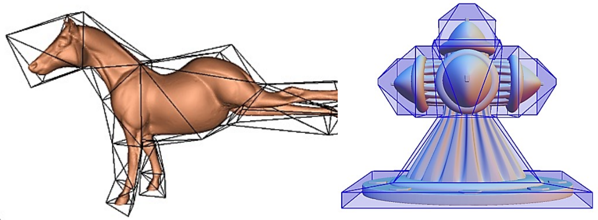
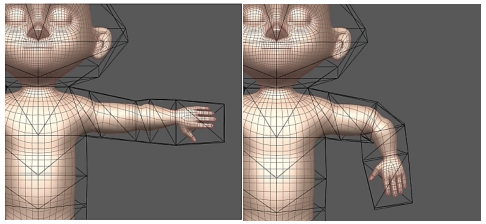


Mean-value coords [Floater 2003; Ju et al. 2005; Thiery et al. 2018]



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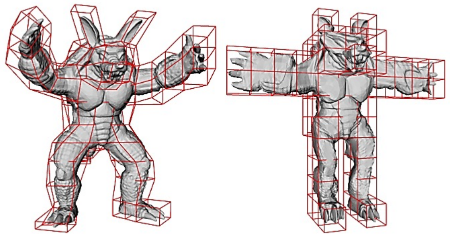

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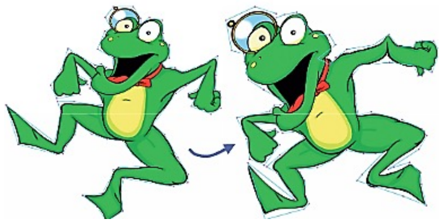

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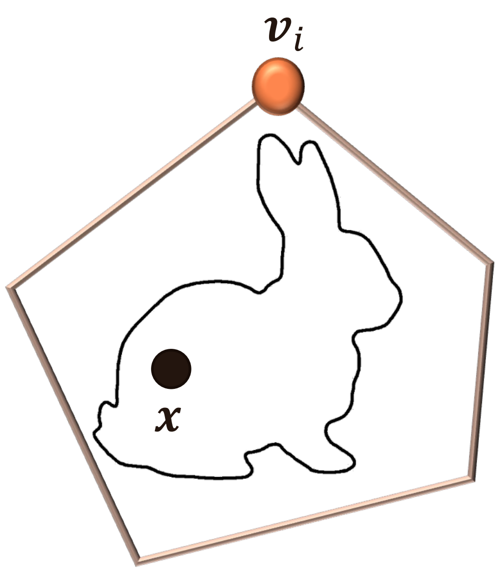

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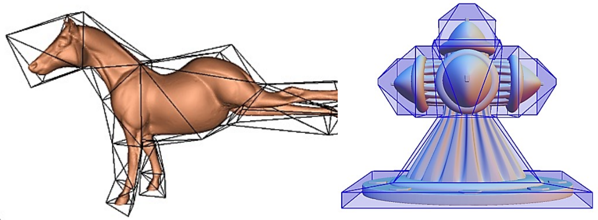

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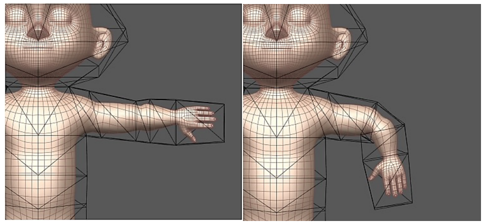

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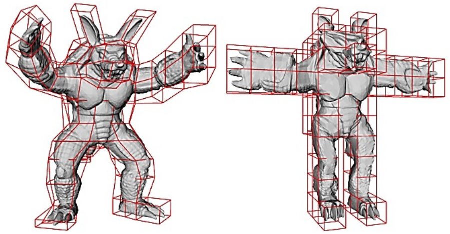

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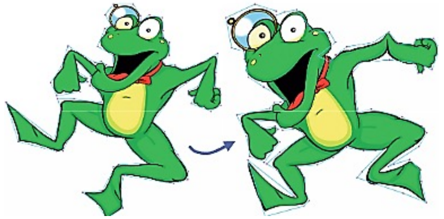

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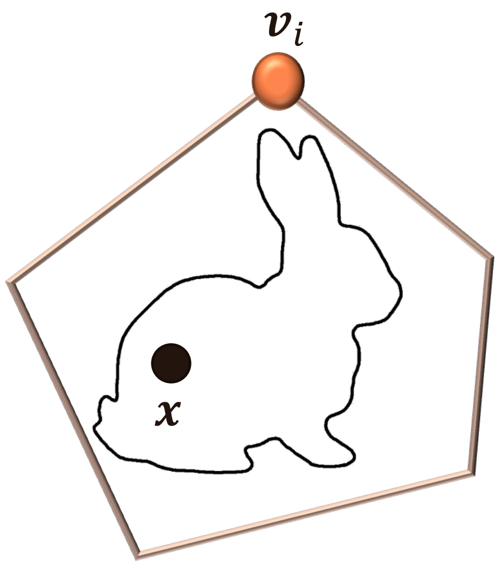

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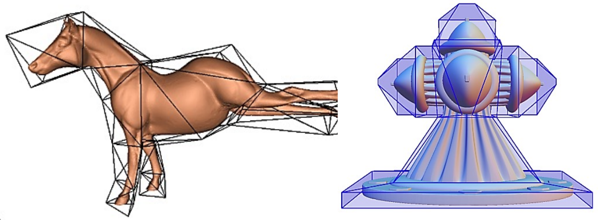

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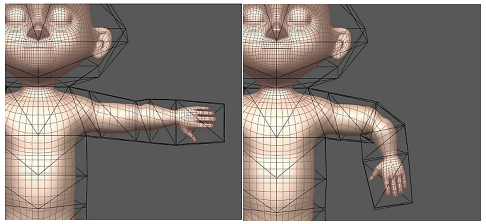

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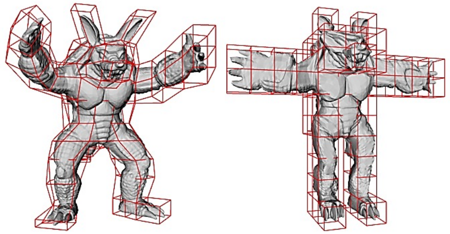

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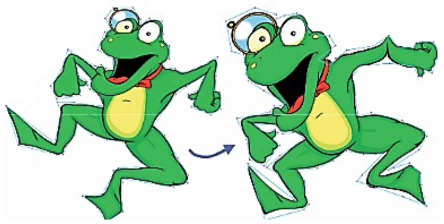

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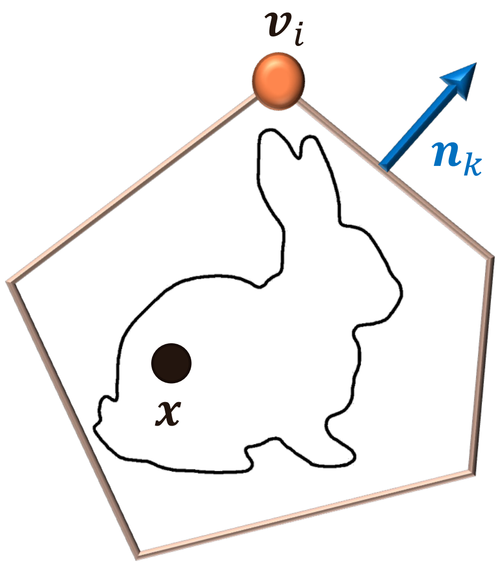

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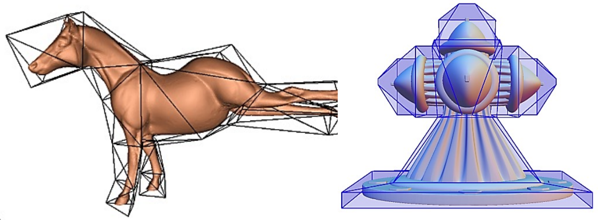

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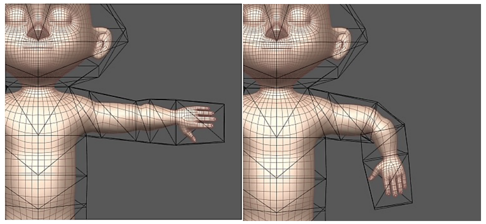

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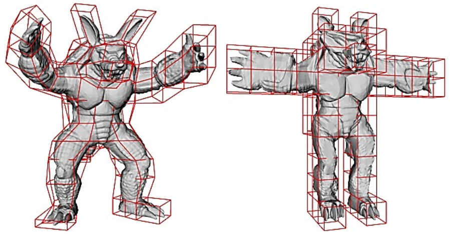

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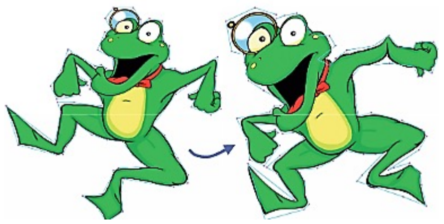

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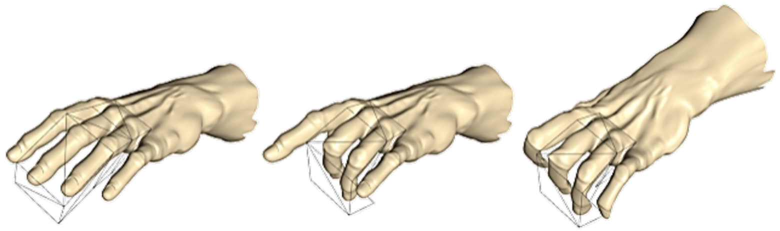
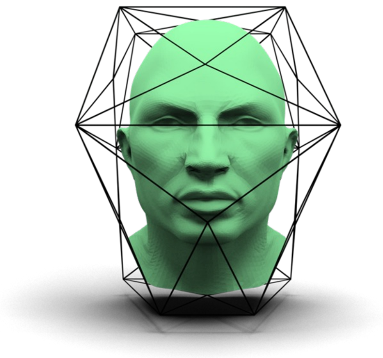
+ Green coordinates [Lipman et al. 2008]



CONTRIBUTIONS

- Inject *elasticity* into cage deformers for fast volumetric deformation



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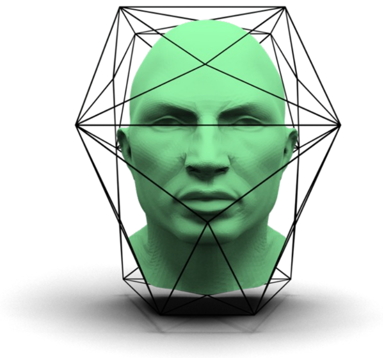

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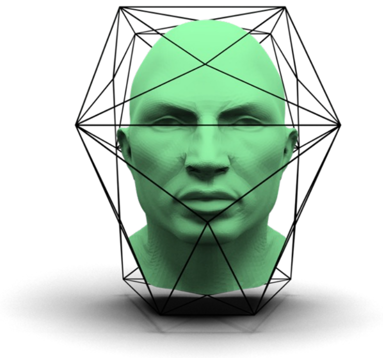

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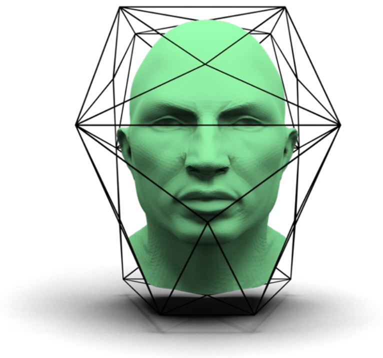

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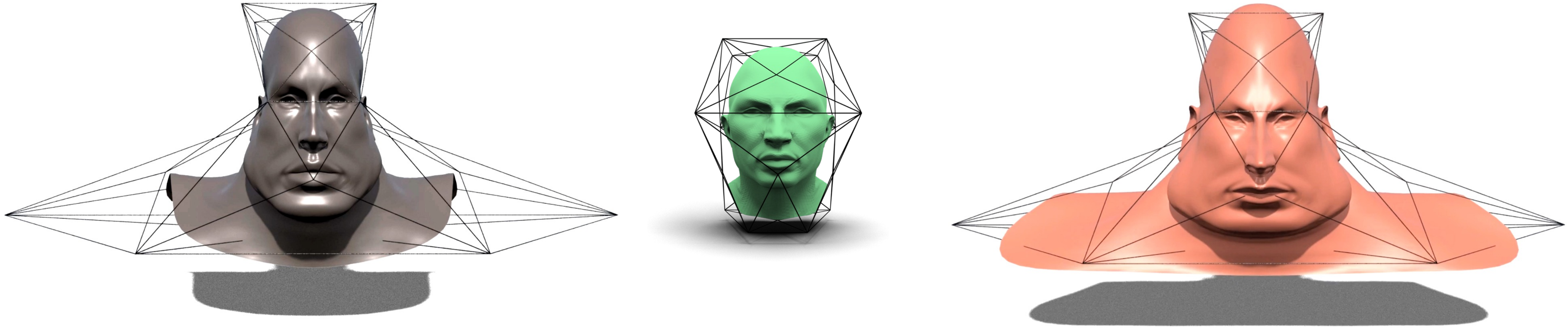
  - Control over **volume change** and **local bulge**



FROM GREEN TO SOMIGLIANA

Green coordinates (GC)

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PDE: $$\Delta \boldsymbol{u} = \boldsymbol{0}$$

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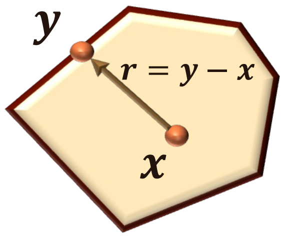
Fundamental solutions:

$$G(\boldsymbol{y}, \boldsymbol{x}) = \begin{cases} -\dfrac{1}{4\pi r}, & d = 3, \\ \dfrac{1}{2\pi}\log(r), & d = 2. \end{cases}$$



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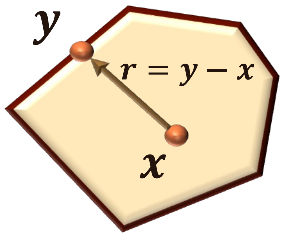

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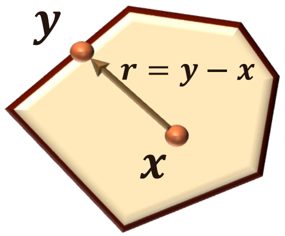

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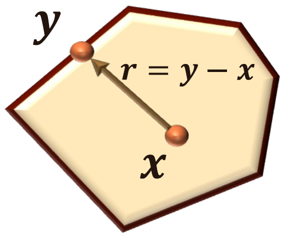

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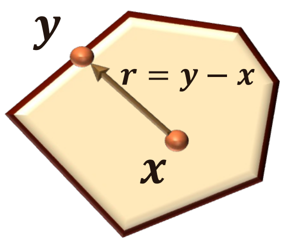

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Somigliana coordinates (SC)

$$\Delta \boldsymbol{u} + \frac{1}{1 - 2\nu} \nabla(\nabla \cdot \boldsymbol{u}) = \boldsymbol{0}$$

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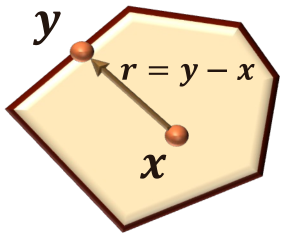

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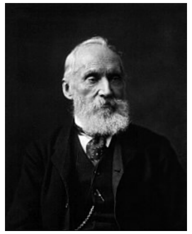

Lord Kelvin

$$\mathcal{K}(\boldsymbol{x},\boldsymbol{y}) = \begin{cases} \frac{a-b}{r}\boldsymbol{I} + \frac{b}{r^3}\boldsymbol{r}\boldsymbol{r}^t, & d=3, \\ (b-a)\log(r)\boldsymbol{I} + \frac{b}{r^2}\boldsymbol{r}\boldsymbol{r}^t, & d=2. \end{cases}$$

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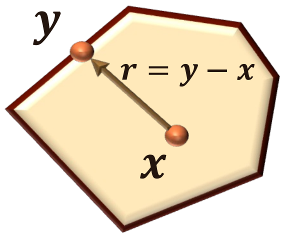

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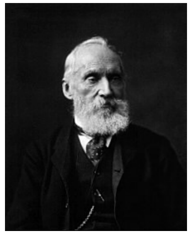
$$\mathcal{K}(\boldsymbol{x},\boldsymbol{y}) = \begin{cases} \frac{a-b}{r}\boldsymbol{I} + \frac{b}{r^3}\boldsymbol{r}\boldsymbol{r}^t, & d = 3, \\ (b-a)\log(r)\boldsymbol{I} + \frac{b}{r^2}\boldsymbol{r}\boldsymbol{r}^t, & d = 2. \end{cases}$$

$$\boldsymbol{u}(\boldsymbol{x}) = \int_{\partial\Omega} [\mathcal{T}(\boldsymbol{y},\boldsymbol{x})\boldsymbol{u}(\boldsymbol{y}) + \mathcal{K}(\boldsymbol{y},\boldsymbol{x})\boldsymbol{\tau}(\boldsymbol{y})] \, d\sigma_y$$


Lord Kelvin


Carlo Somigliana

FROM GREEN TO SOMIGLIANA

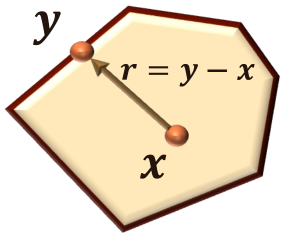
Green coordinates (GC)

PDE:

$$\Delta \boldsymbol{u} = \boldsymbol{0}$$

Fundamental solutions:

$$G(\boldsymbol{y},\boldsymbol{x}) = \begin{cases} -\dfrac{1}{4\pi r}, & d = 3, \\ \dfrac{1}{2\pi}\log(r), & d = 2. \end{cases}$$



Boundary reformulation:

$$\boldsymbol{u}(\boldsymbol{x}) = \int_{\partial\Omega} [\nabla_n G(\boldsymbol{y},\boldsymbol{x}) \boldsymbol{u}(\boldsymbol{y}) - G(\boldsymbol{y},\boldsymbol{x}) \nabla_n \boldsymbol{u}(\boldsymbol{y})] \, d\sigma_y$$

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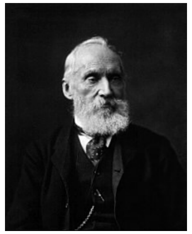

Lord Kelvin

Carlo Somigliana

$$\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{x}$$

$$\{\phi_i(\boldsymbol{x}), \psi_k(\boldsymbol{x})\} \in \mathbf{R}$$

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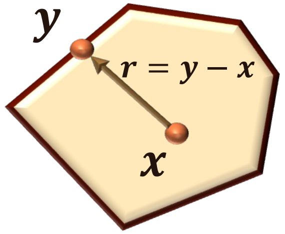

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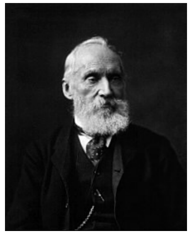

Lord Kelvin

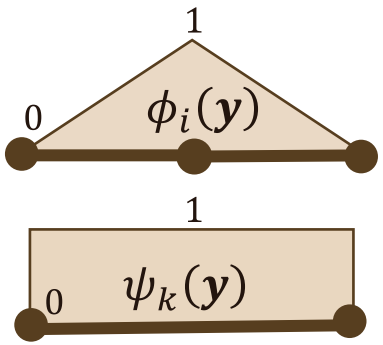

Carlo Somigliana

$$\boldsymbol{u}(\boldsymbol{x}) = \boldsymbol{x}$$

$$\{T_i(\boldsymbol{x}), K_k(\boldsymbol{x})\} \in \mathbf{R}^{d\times d}$$

Compute SC w.r.t. a triangulated cage

$$\begin{cases} T_i(\boldsymbol{x}) = \int_{\partial\Omega} \mathcal{T}(\boldsymbol{y}, \boldsymbol{x})\phi_i(\boldsymbol{y})\mathrm{d}\sigma_{\boldsymbol{y}}, \\ K_k(\boldsymbol{x}) = \int_{\partial\Omega} \mathcal{K}(\boldsymbol{y}, \boldsymbol{x})\psi_k(\boldsymbol{y})\mathrm{d}\sigma_{\boldsymbol{y}}. \end{cases}$$



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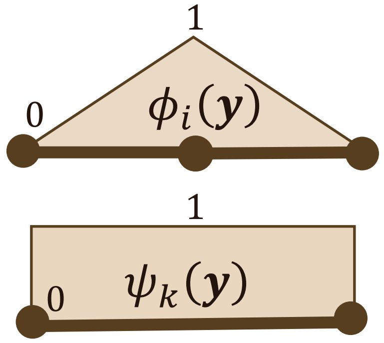

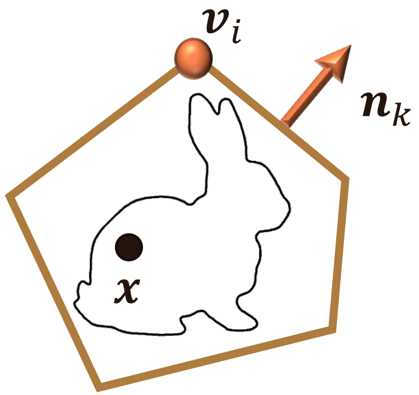

$$\boldsymbol{x} = \sum_i T_i(\boldsymbol{x})\boldsymbol{v}_i + \sum_k K_k(\boldsymbol{x})(c\boldsymbol{n}_k)$$

SOMIGLIANA COORDINATES

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T_i and K_k as matrix functions of Poisson ratio ν

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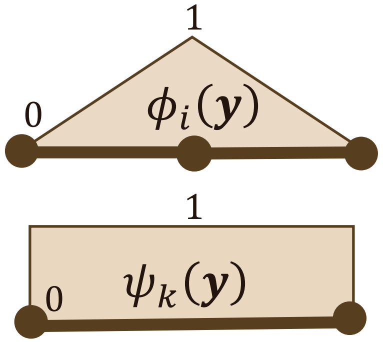

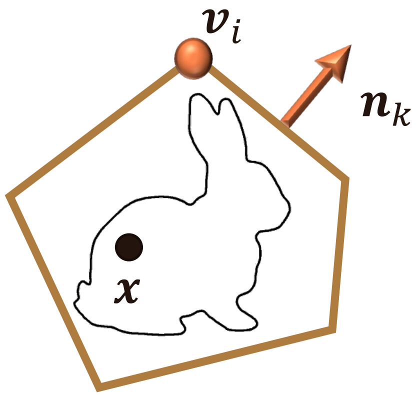

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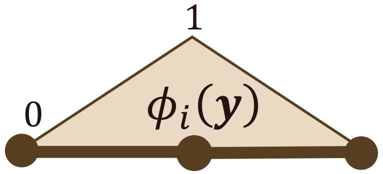

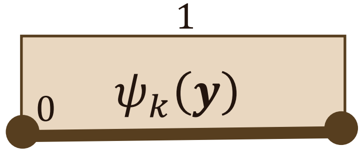

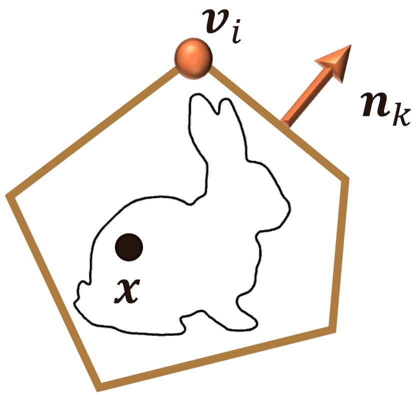

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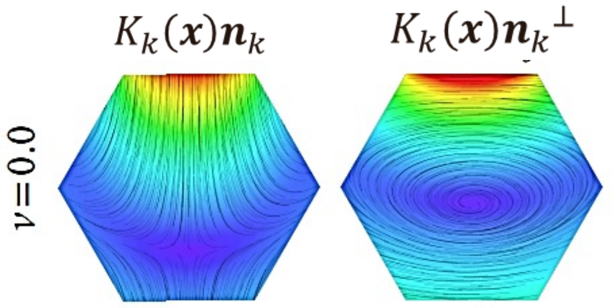

SOMIGLIANA COORDINATES

SIGGRAPH 2023
LOS ANGELES+ 6-10 AUG

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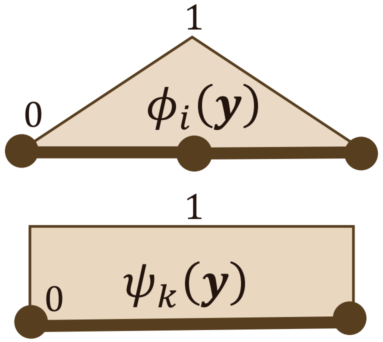

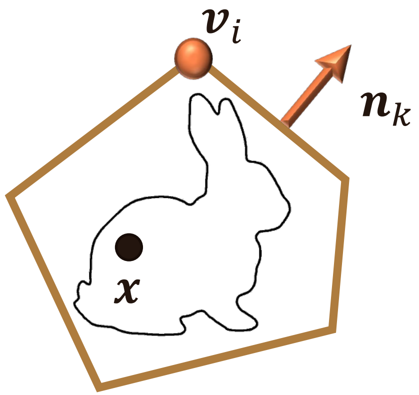

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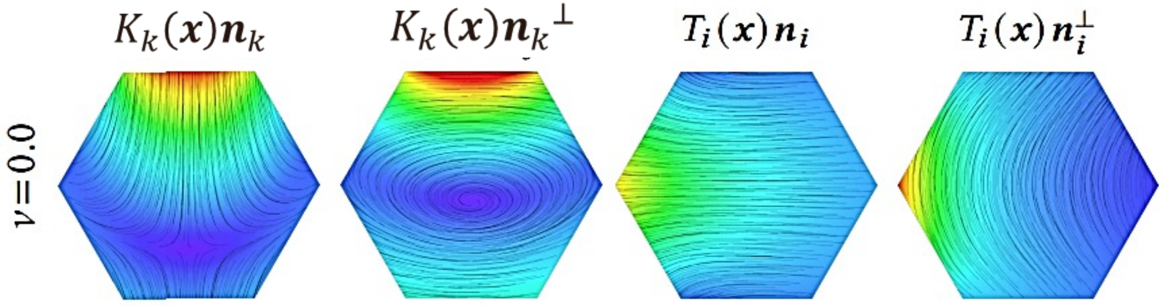

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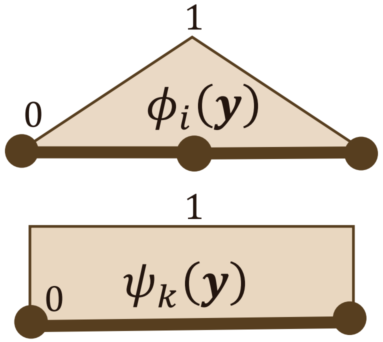

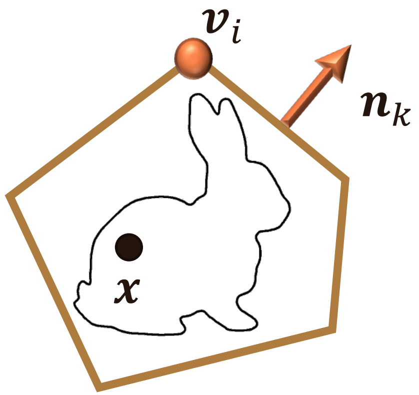

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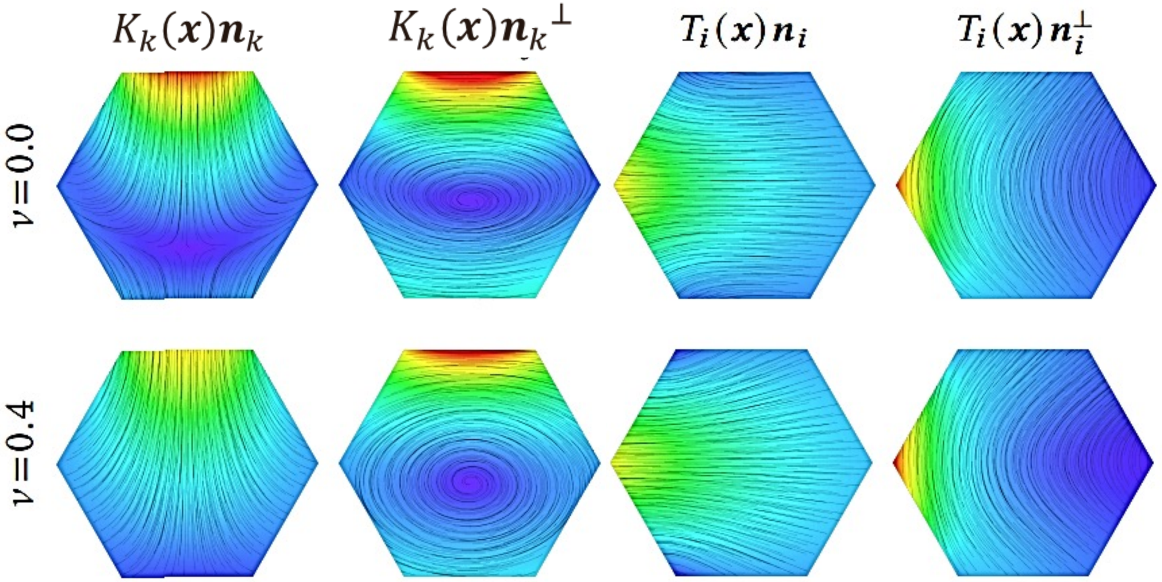
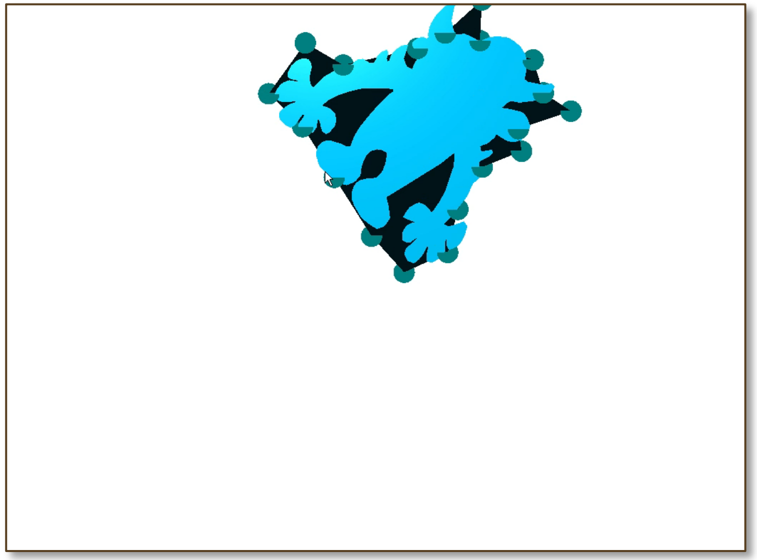


COROTATIONAL FORMULATION

$T_i(\boldsymbol{x})$ and $K_k(\boldsymbol{x})$ are not rotationally invariant



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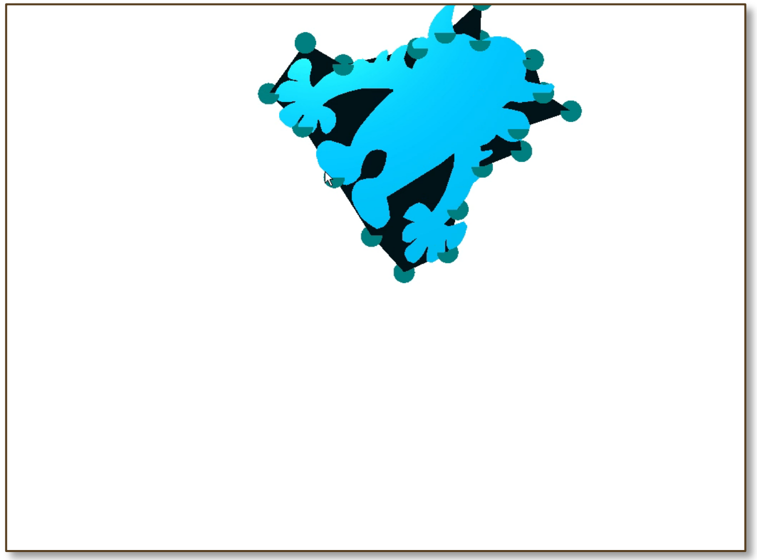

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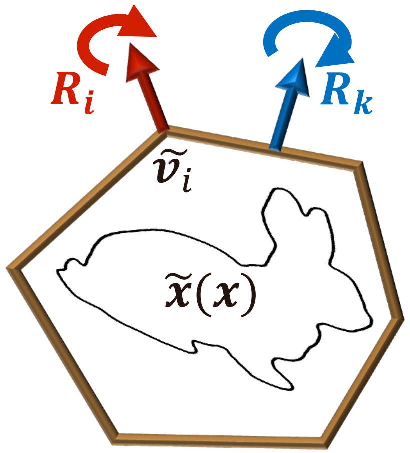

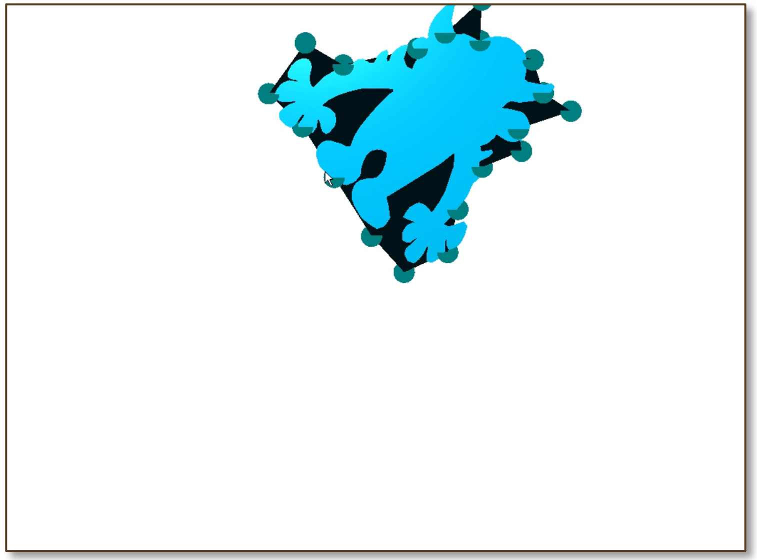

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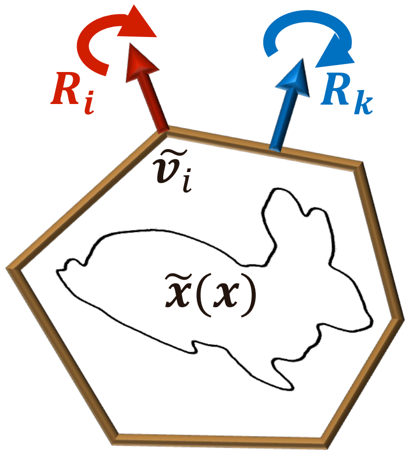
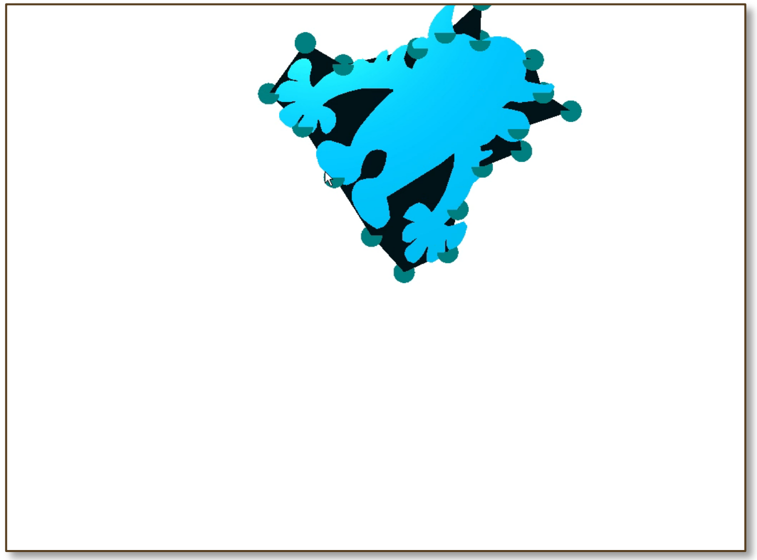


$$\tilde{\boldsymbol{\tau}}_k = s_k R_k \boldsymbol{n}_k$$

$$= \left[\frac{2(1-\nu)}{1-2\nu}\eta_k + \frac{2\nu(d-1)}{1-2\nu}\lambda_k\right] R_k \boldsymbol{n}_k$$



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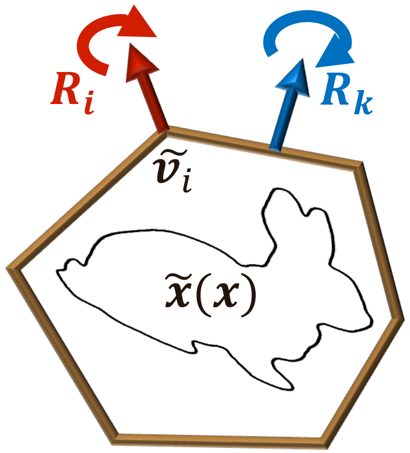

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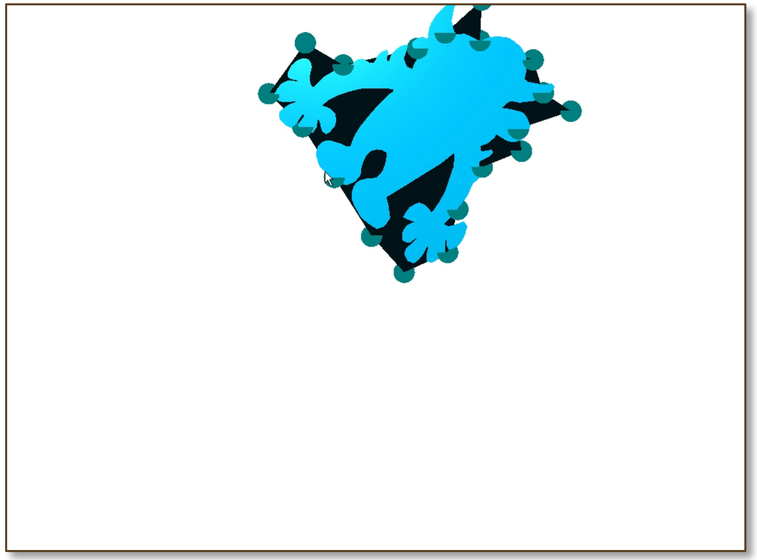
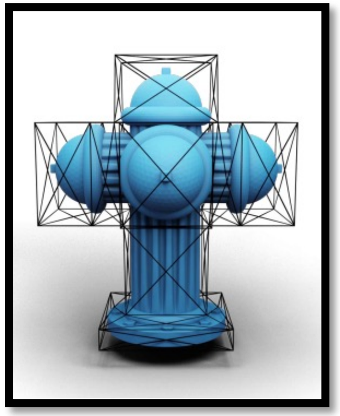
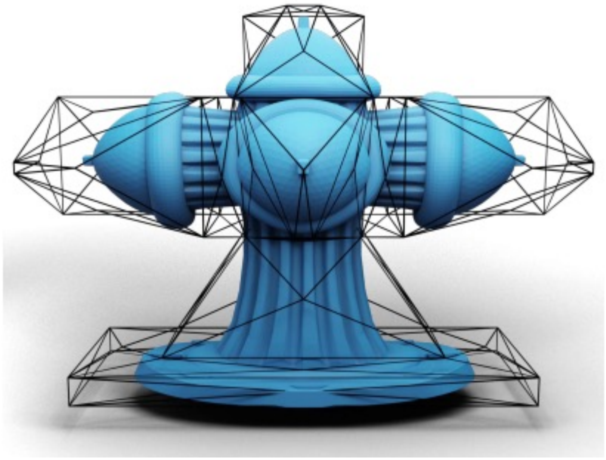


Estimate $\{R_k, \lambda_k, \eta_k\}$ for each boundary facet

GLOBAL VS. LOCAL ROTATION & TANGENT STRETCHES



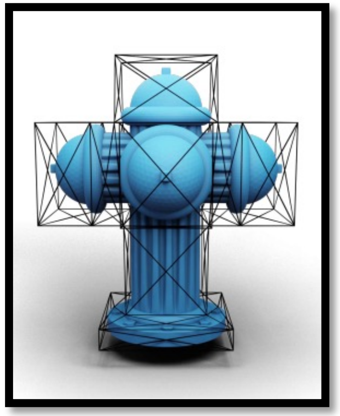


Rest pose

Global variant

$R_k = R_{global}, \lambda_k = \lambda_{global}$ from the optimal similarity transformation

GLOBAL VS. LOCAL ROTATION & TANGENT STRETCHES



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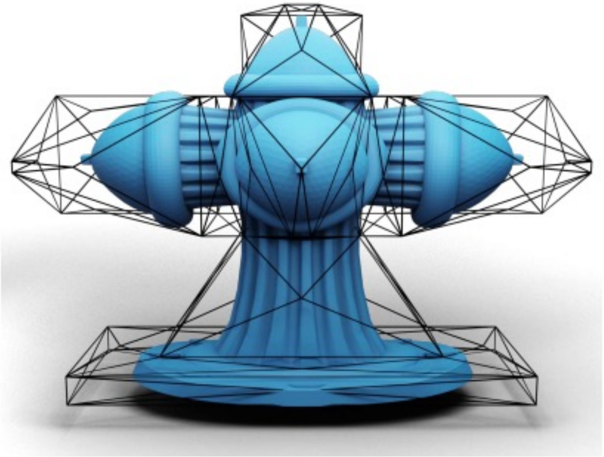

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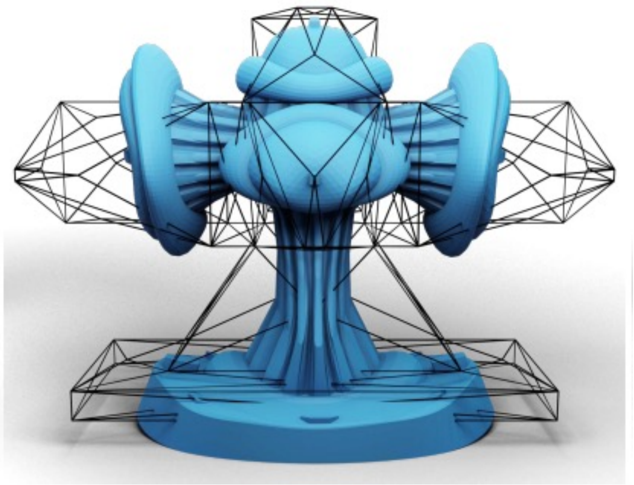
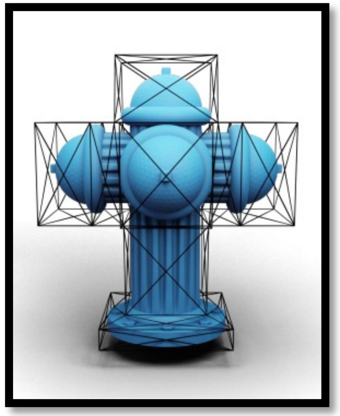
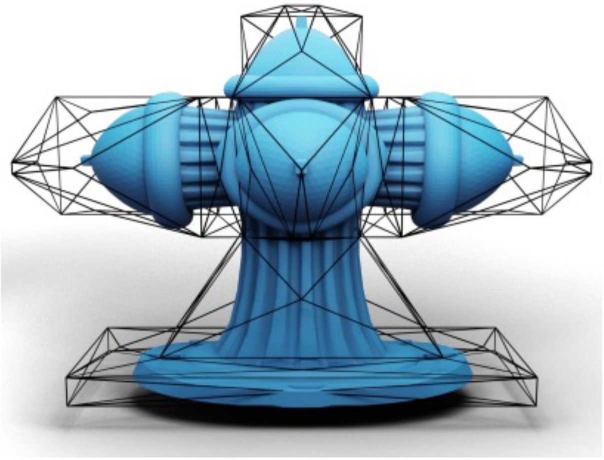


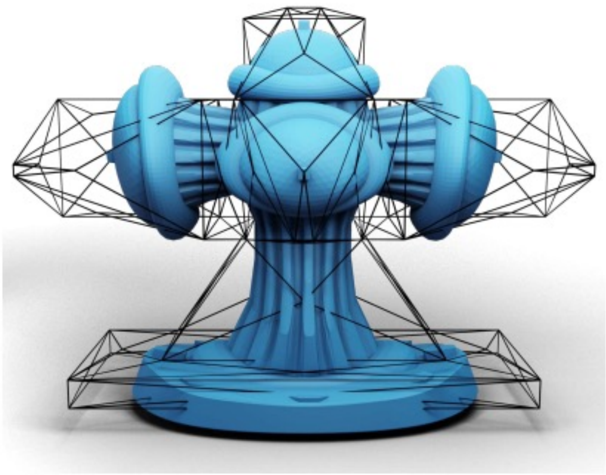
Local variant

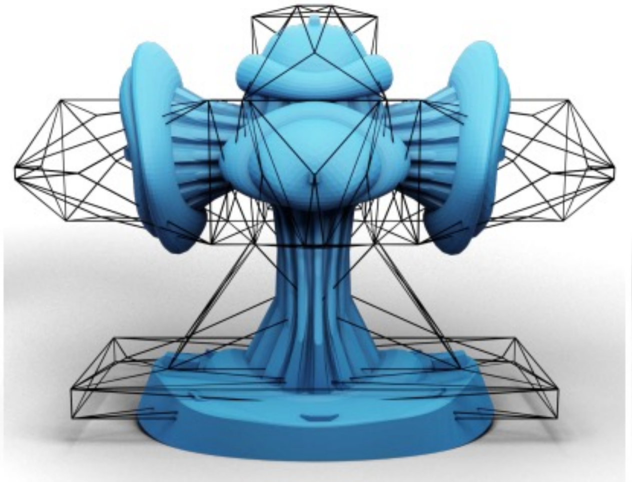
R_k and λ_k are decuded on per facet basis

GLOBAL VS. LOCAL ROTATION & TANGENT STRETCHES









Rest pose

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In between

blend global and local R_k, λ_k

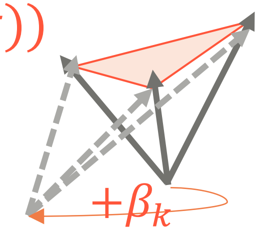
Local variant

R_k and λ_k are decuded on per facet basis

CURVATURE-BASED NORMAL STRETCHES

- Normal stretching factor η_k for each cage facet
  - No information about out-of-plane deformation
  - E.g., account for curvature change for local bulging

$$\eta_k = \lambda_k \exp(\gamma \beta_k / (2^{d-1}\pi))$$



  - Compute on-the-fly

Small γ

Large γ

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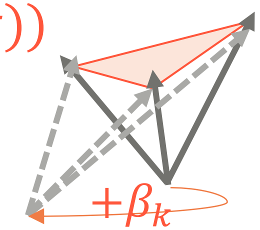

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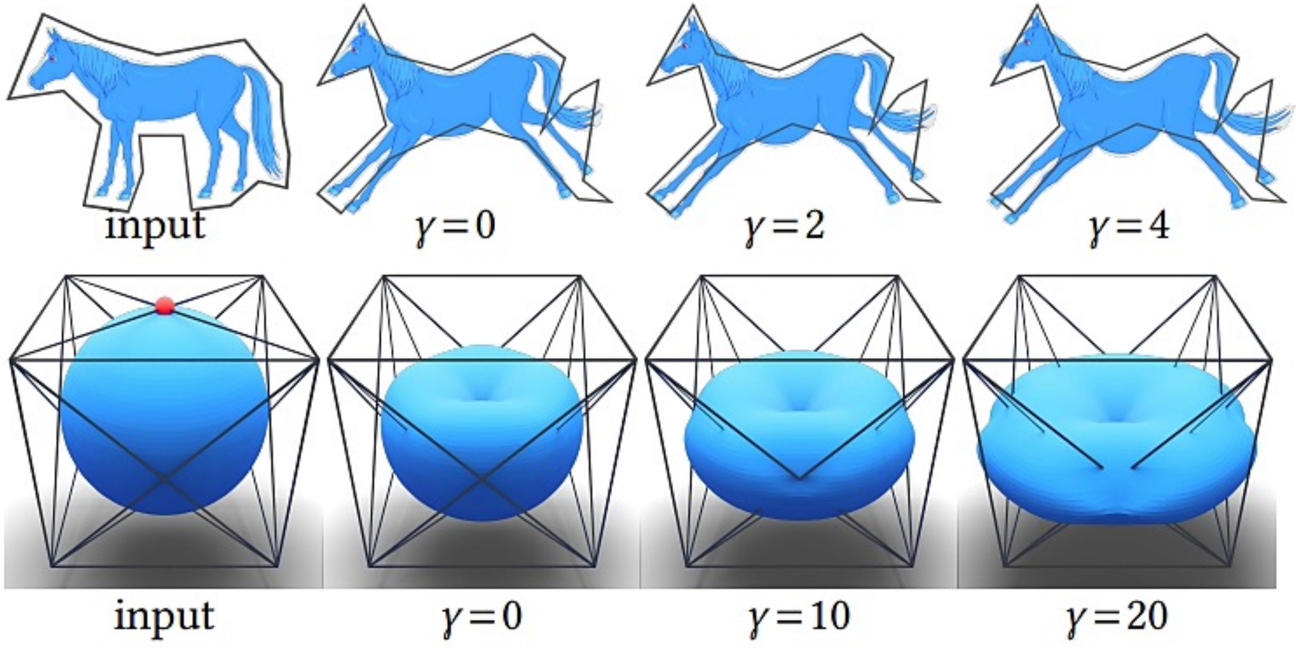

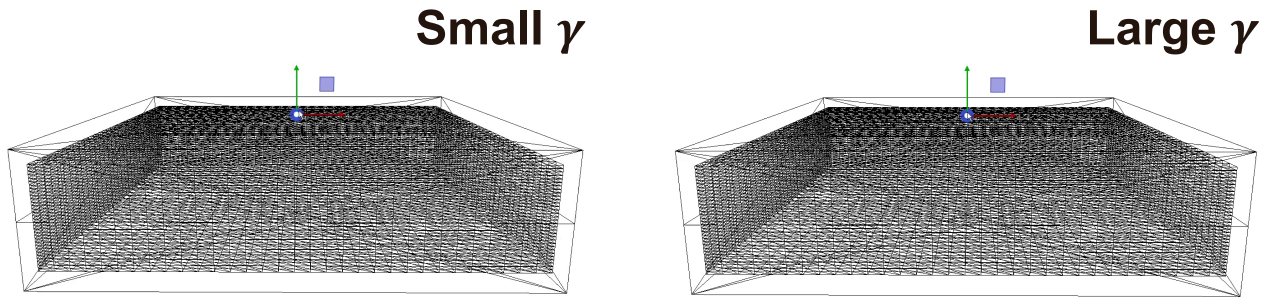

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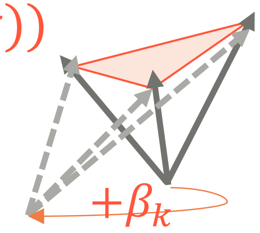
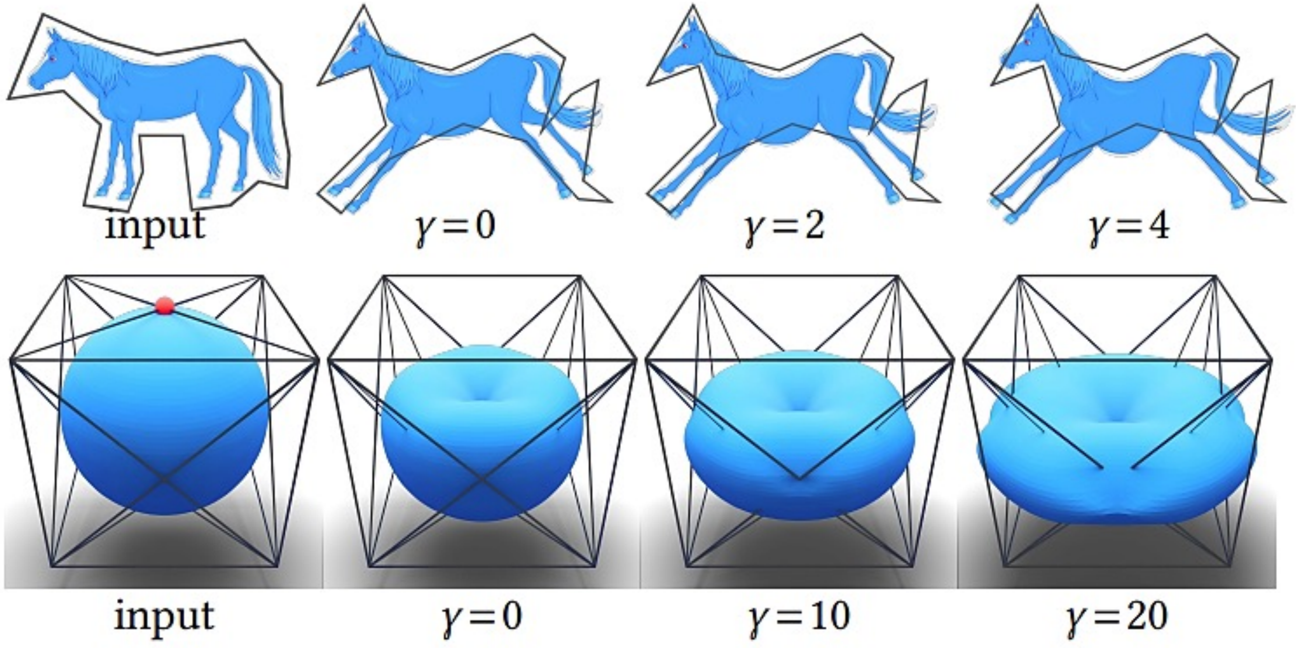
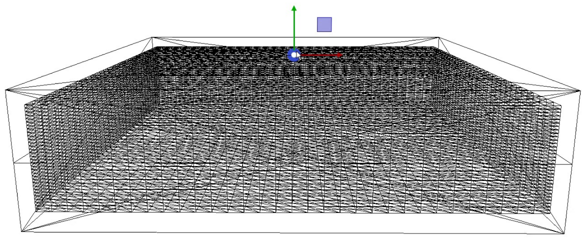


- Our choice of R_j, λ_j, η_j keeps the deformation invariant under similarity transformations

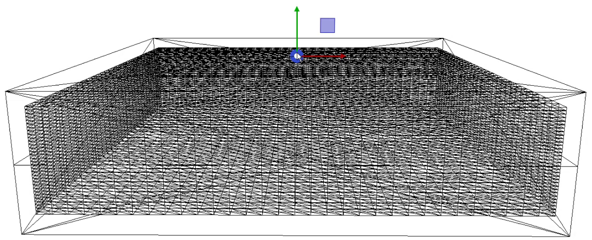
$$\tilde{\boldsymbol{x}}(sR\boldsymbol{x} + \boldsymbol{t}) = sR\tilde{\boldsymbol{x}}(\boldsymbol{x}) + \boldsymbol{t}$$



Small γ

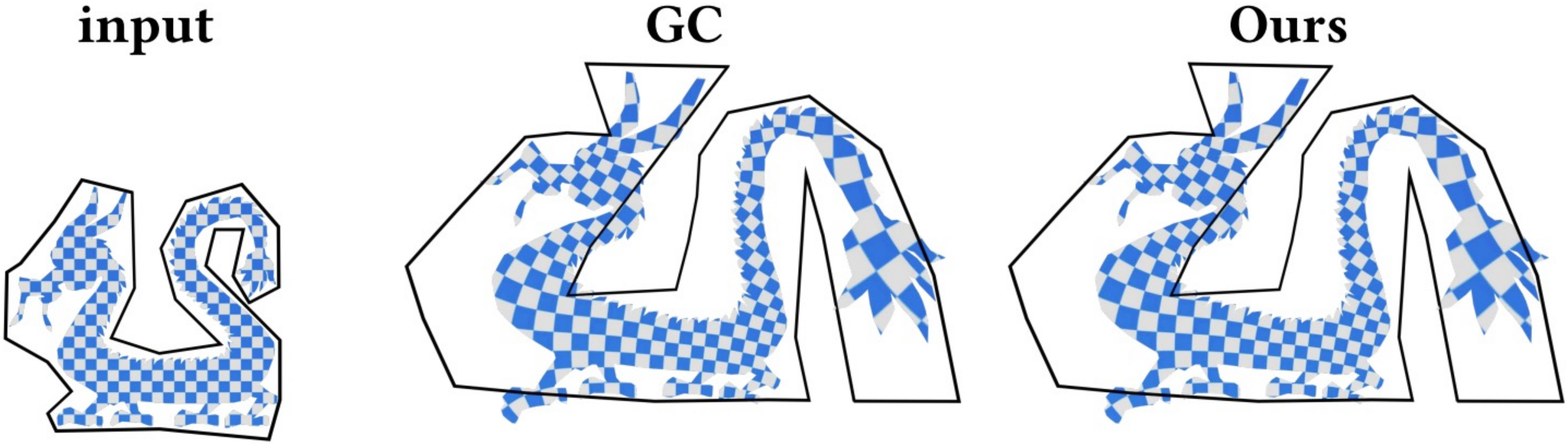


Large γ



RELATION TO GREEN'S COORDINATES

- SC is equivalent to GC in 2D, for $\nu = \infty$, and $\gamma = 0$



RELATION TO GREEN'S COORDINATES

- SC is equivalent to GC in 2D, for $\nu = \infty$, and $\gamma = 0$

2D SC deformation with $\nu = \infty,\ \gamma = 0$

$$\widetilde{\boldsymbol{x}}(\boldsymbol{x}) = \frac{1}{2\pi}\sum_e \int_0^{L_e} \left(\frac{\boldsymbol{r}^t\boldsymbol{n}}{r^2}\boldsymbol{I} + \frac{1}{r^2}(\boldsymbol{n}\boldsymbol{r}^t - \boldsymbol{r}\boldsymbol{n}^t)\right)\widetilde{\boldsymbol{y}}\, d\sigma_{\boldsymbol{y}}$$

$$= \frac{1}{2\pi}\sum_e \int_0^{L_e} \frac{1}{r^2}\left\{\begin{pmatrix} r_1n_1+r_2n_2 & 0 \\ 0 & r_1n_1+r_2n_2 \end{pmatrix} + \begin{pmatrix} 0 & r_2n_1-r_1n_2 \\ r_1n_2-r_2n_1 & 0 \end{pmatrix}\right\}\widetilde{\boldsymbol{y}}\, d\sigma_{\boldsymbol{y}}.$$

Expressed in complex numbers

$$\frac{1}{2\pi}\int_0^{L_e} \frac{r_1n_1+r_2n_2+\mathrm{i}(r_1n_2-r_2n_1)}{r^*r}\widetilde{y}\, d\sigma_y$$

$$= \frac{1}{2\pi}\int_0^{L_e} \frac{(r_1-\mathrm{i}r_2)(n_1+\mathrm{i}n_2)}{r^*r}\widetilde{y}\, d\sigma_y$$

$$= \frac{1}{2\pi}\int_0^{L_e} \frac{r^*n}{r^*r}\widetilde{y}\, d\sigma_y = \frac{1}{2\pi\mathrm{i}}\int_0^{L_e} \frac{\widetilde{y}}{r}\mathrm{i}\cdot n\, d\sigma_y$$

$$= \frac{1}{2\pi\mathrm{i}}\int_L \frac{\widetilde{y}}{r}\, dy,$$

Through Cauchy integral formula

$$g_{S,f}(z) = \sum_{j=1}^{n} C_j(z) f_j$$

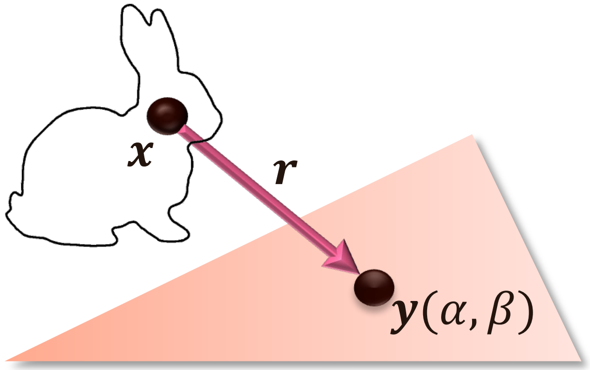
$$C_j(z) = \frac{1}{2\pi i}\left(\frac{B_{j+1}(z)}{A_{j+1}}\log\left(\frac{B_{j+1}(z)}{B_j(z)}\right) - \frac{B_{j-1}(z)}{A_j}\log\left(\frac{B_j(z)}{B_{j-1}(z)}\right)\right)$$

Cauchy-Green complex barycentric coordinates

Theorem 3: Lipman's 2D Green coordinates [LLCO08] are identical to discrete Cauchy coordinates.

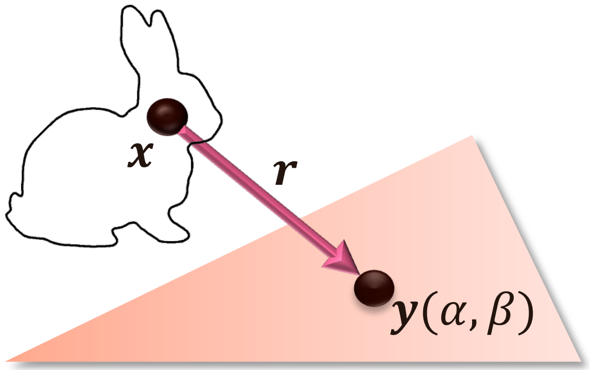
[Weber et at. 2009]

IMPLEMENTATION



$$K_k(\boldsymbol{x}) = \int_{\Delta_k} \mathcal{K}(\boldsymbol{y}, \boldsymbol{x}) \mathrm{d}\sigma_{\boldsymbol{y}} = 2|\Delta_k| \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \, \mathcal{K}(\boldsymbol{y}(\alpha, \beta), \boldsymbol{x})$$

IMPLEMENTATION



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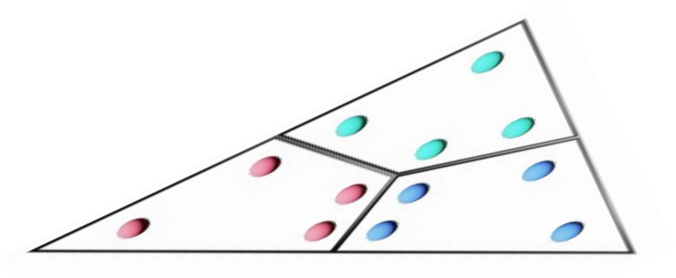
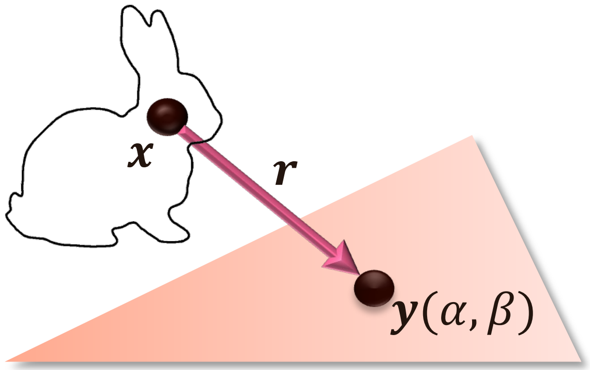
$$\approx$$



$$\{(\alpha_j, \beta_j), w_j\}$$

$$2|\Delta_k| \sum_j w_j \left(\frac{a-b}{r(\alpha_j,\beta_j)} \boldsymbol{I} + \frac{b}{r^3(\alpha_j,\beta_j)} \boldsymbol{r}(\alpha_j,\beta_j) \boldsymbol{r}^t(\alpha_j,\beta_j) \right)$$

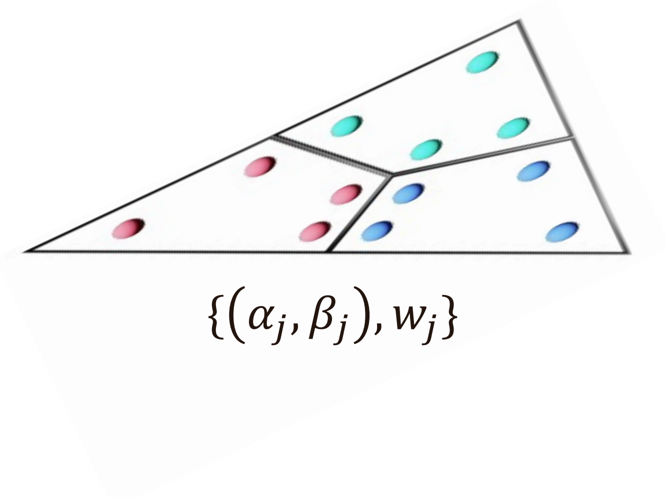
IMPLEMENTATION



$$K_k(\boldsymbol{x}) = \int_{\Delta_k} \mathcal{K}(\boldsymbol{y},\boldsymbol{x})\mathrm{d}\sigma_{\boldsymbol{y}} = 2|\Delta_k| \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta\, \mathcal{K}(\boldsymbol{y}(\alpha,\beta),\boldsymbol{x})$$

- #query points: **39k**
- #quadratures per face: **7500**
- #cage faces: **184**

$\approx$ Coord. computation time: **1.5s**



$$2|\Delta_k| \sum_j w_j \left(\frac{a-b}{r(\alpha_j,\beta_j)} \boldsymbol{I} + \frac{b}{r^3(\alpha_j,\beta_j)} \boldsymbol{r}(\alpha_j,\beta_j)\boldsymbol{r}^t(\alpha_j,\beta_j) \right)$$

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RESULTS

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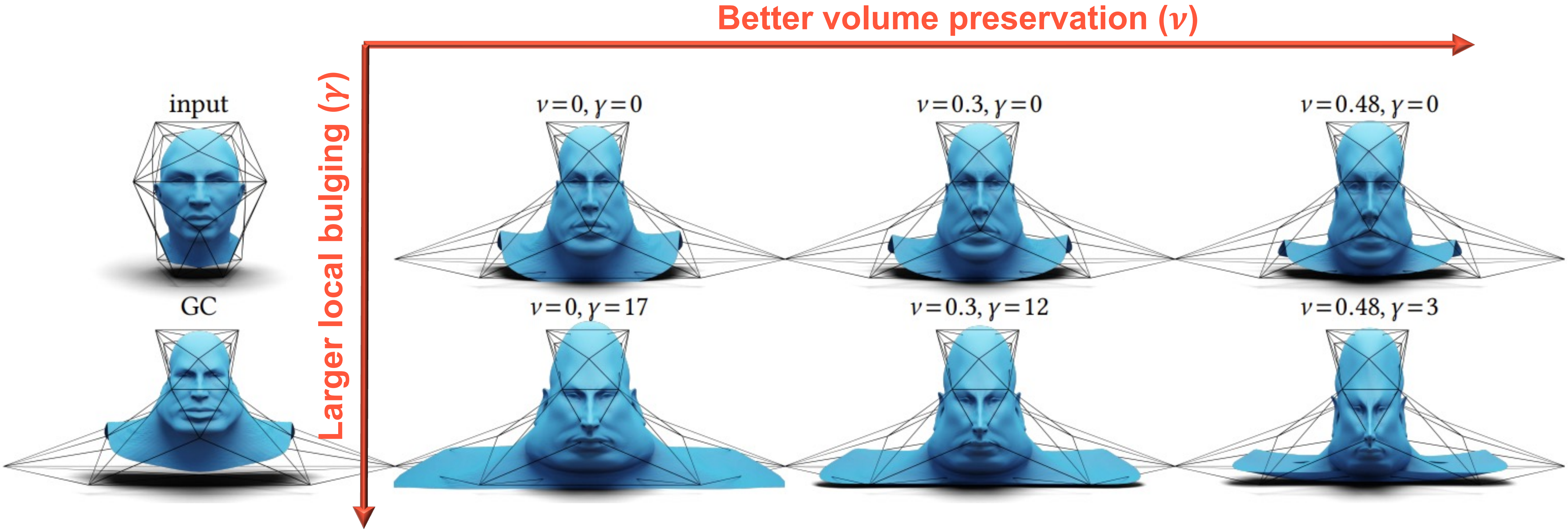
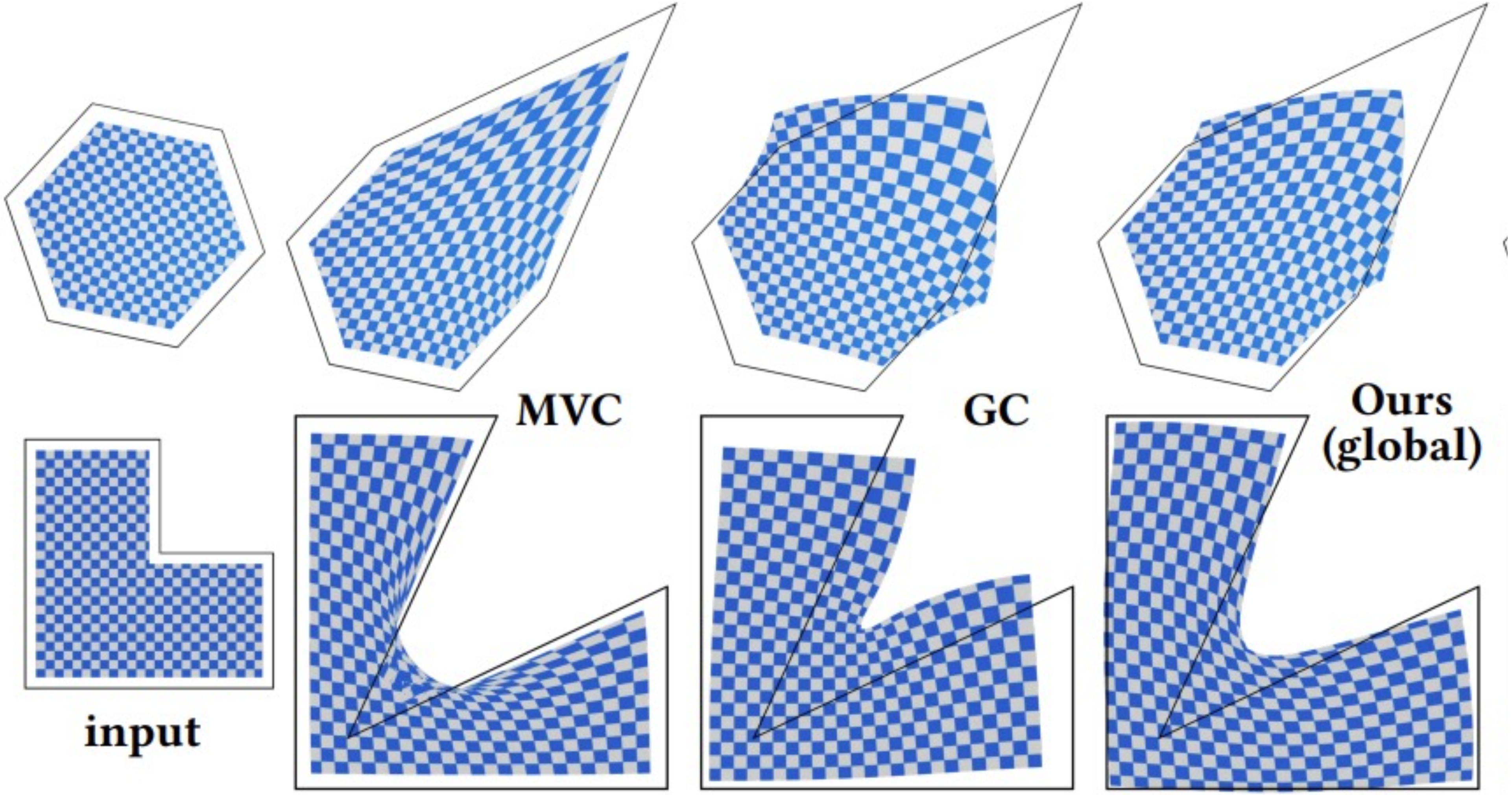
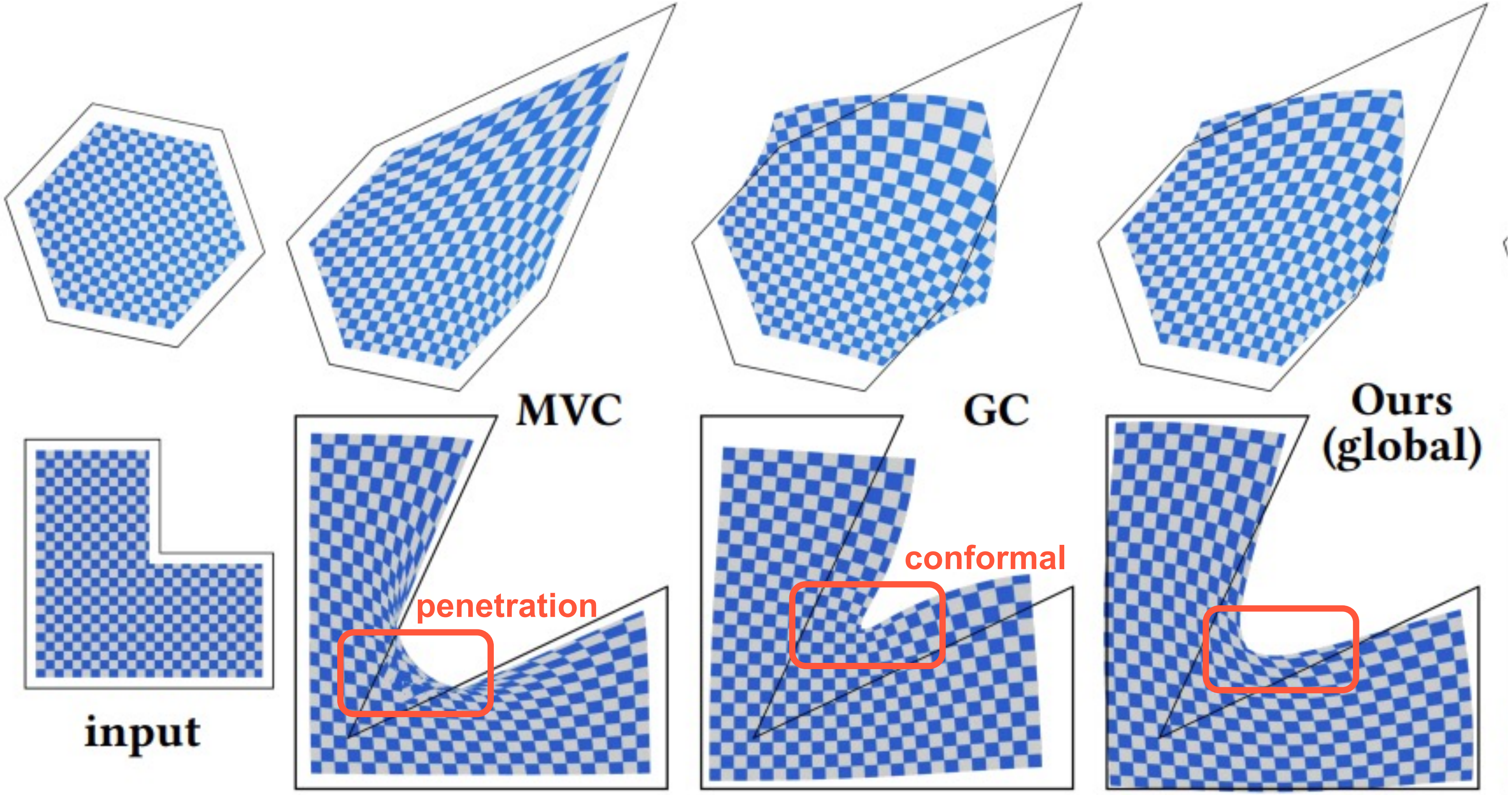
VOLUME PRESERVING VS. LOCAL BULGING



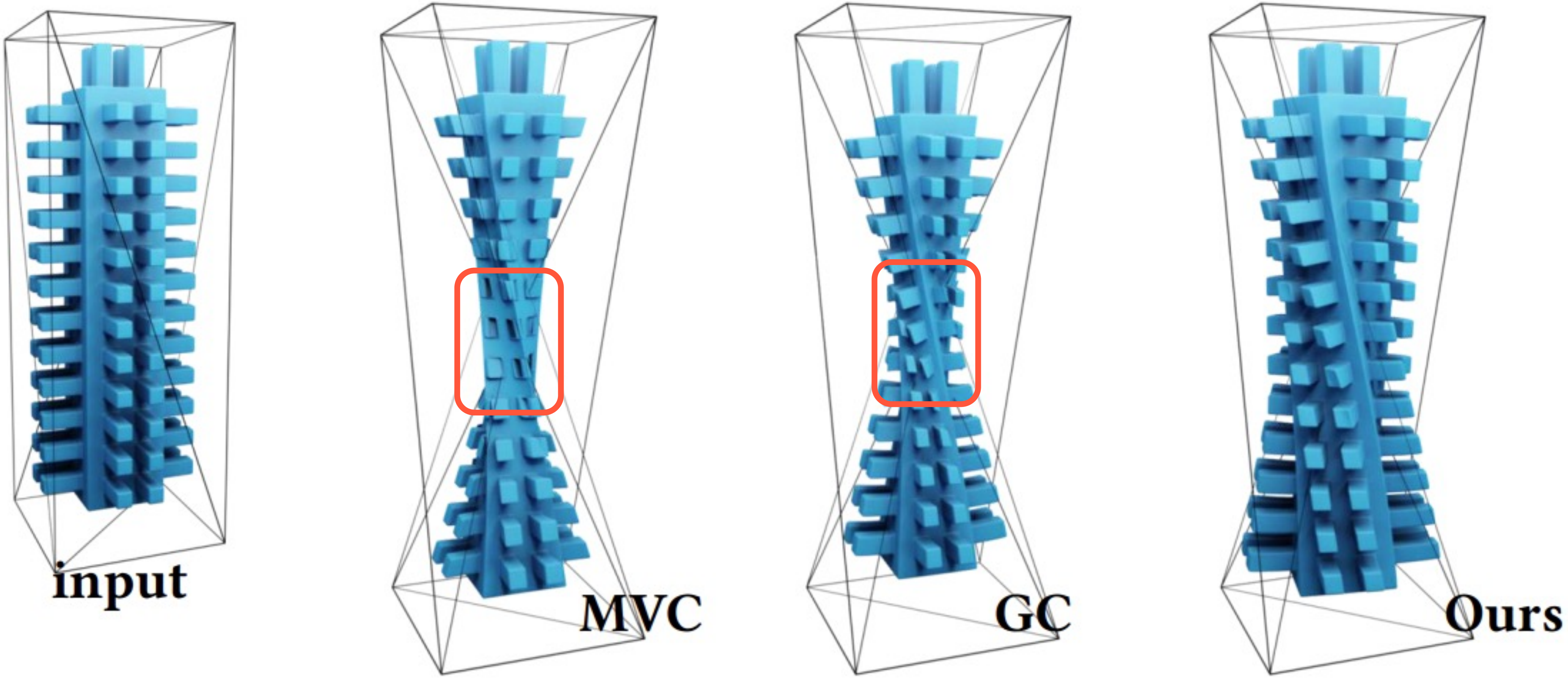
2D COMPARISONS



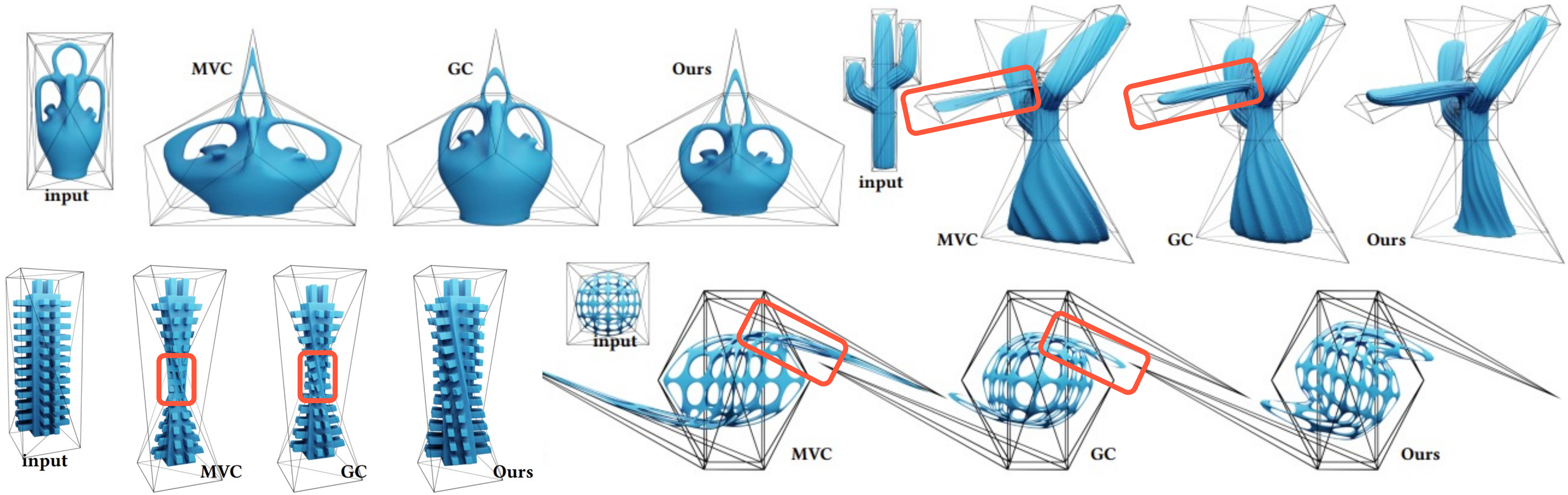
2D COMPARISONS



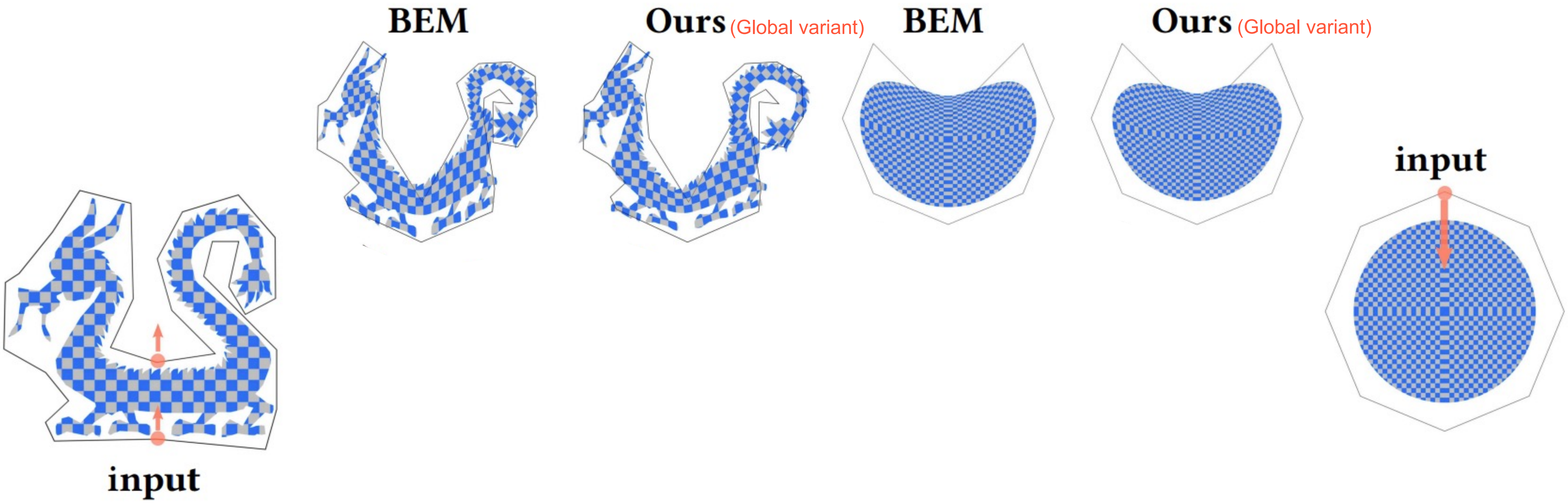
3D COMPARISONS



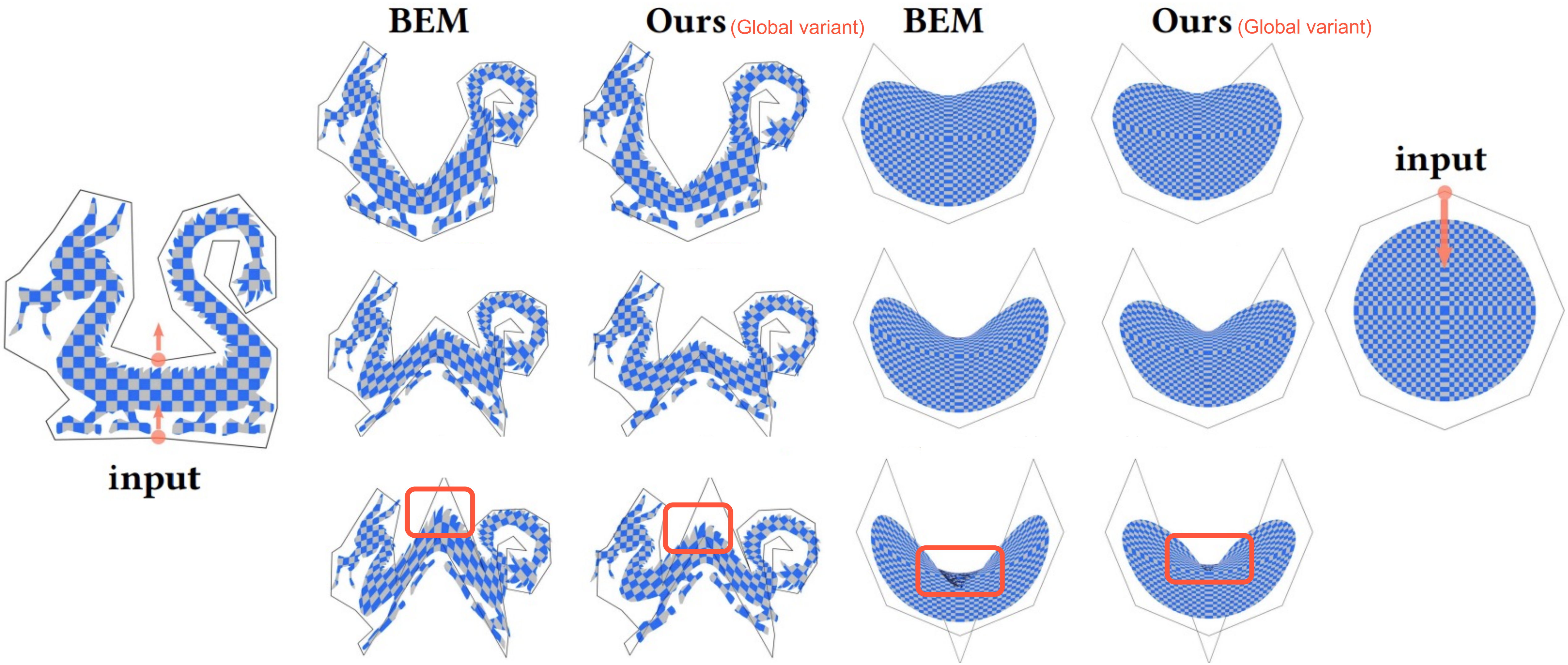
3D COMPARISONS



COMPARISONS

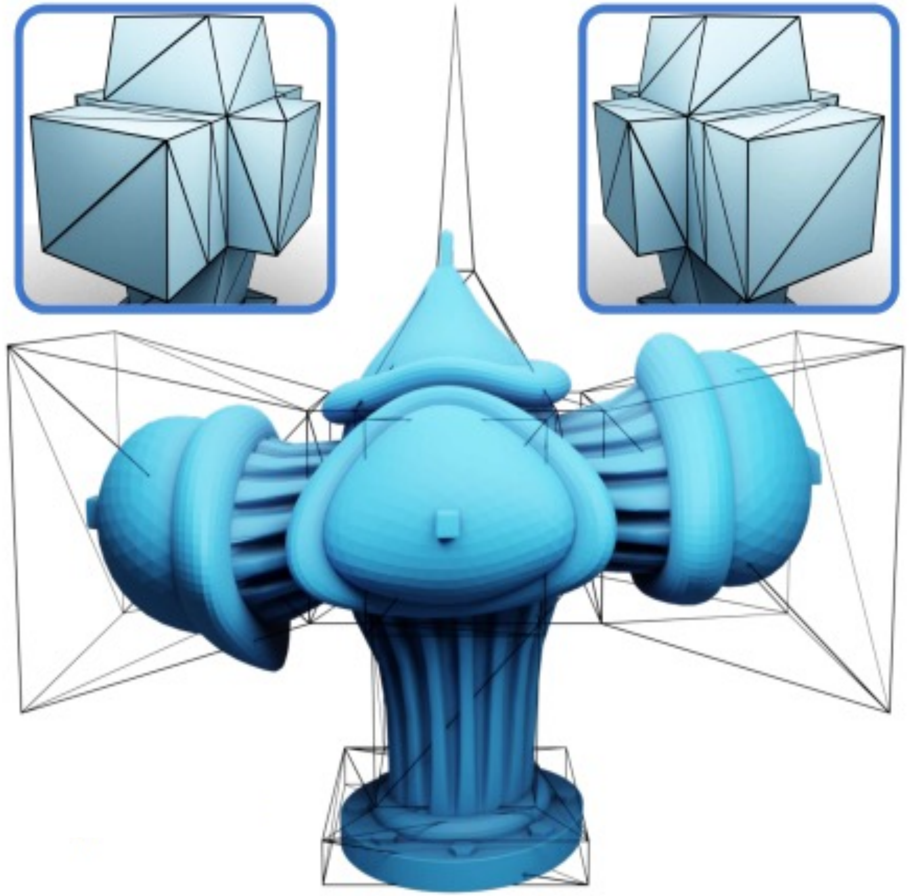


COMPARISONS



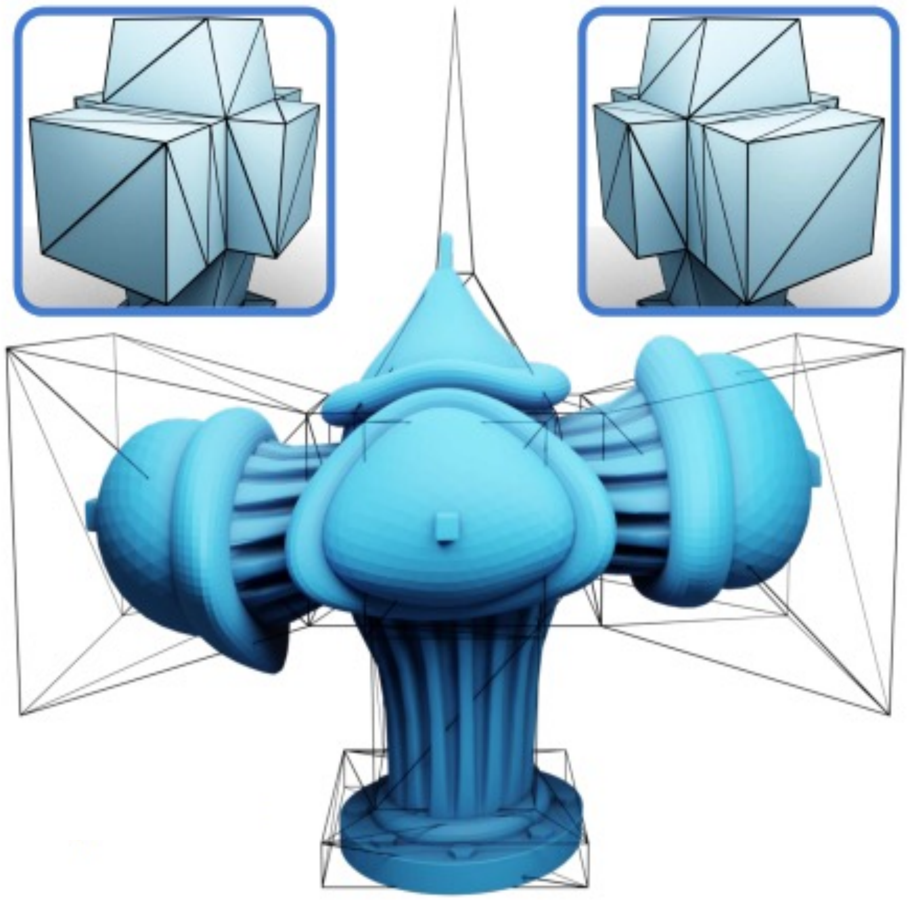
FUTURE WORKS

- Cage triangulation may break the symmetry
  - Compute SC on quad meshes
  - Adopt other bulging factors less sensitive to the triangulation



FUTURE WORKS

- Cage triangulation may break the symmetry
  - Compute SC on quad meshes
  - Adopt other bulging factors less sensitive to the triangulation
- Accelerate SC computations
  - Adaptive quadrature rules for far- and near-field evaluations
  - Derive closed-form expressions



FUTURE WORKS

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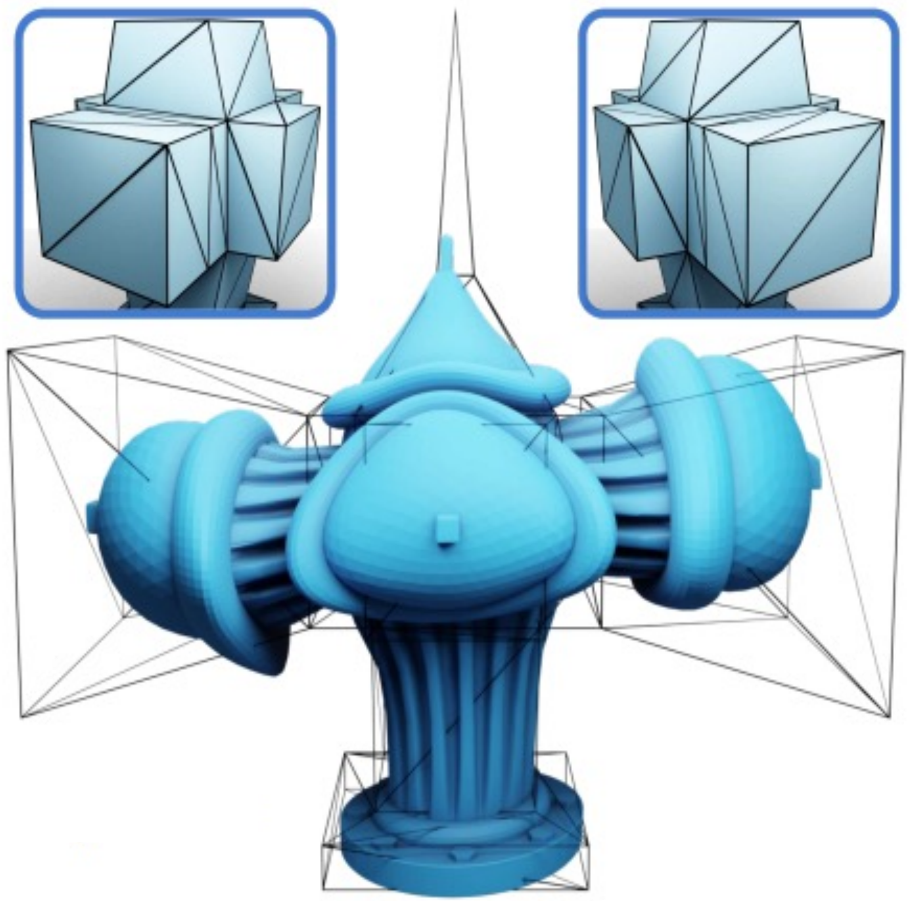
  - Derive closed-form expressions
- Space-time cages for real-time animation editing



Code available here