Lightning-fast Boundary Element Method

Jiong Chen[1], Florian Schäfer[2], Mathieu Desbrun[1,3]

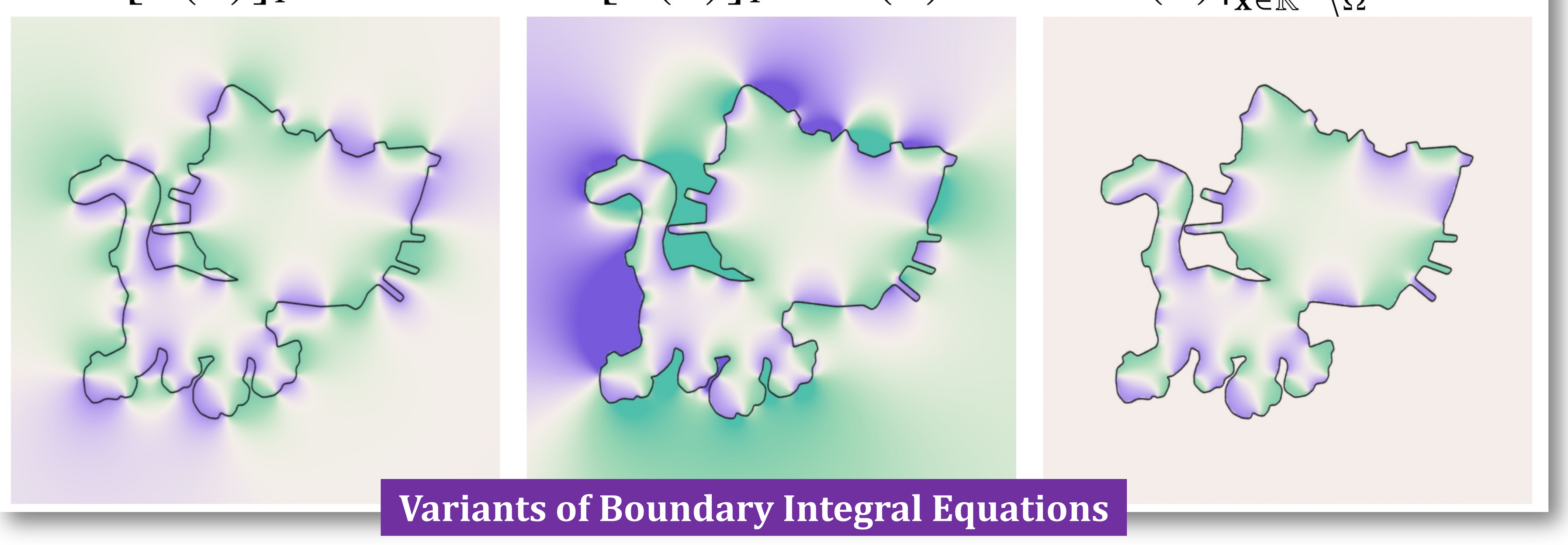
[1]Inria [2]Georgia Tech [3]Ecole Polytechnique

Representing solutions through boundary potentials

$$\forall \mathbf{x} \in \mathbb{R}^d \backslash \Gamma,\ u(\mathbf{x}) = \int_\Gamma \frac{\partial G(\mathbf{x},\mathbf{y})}{\partial \mathbf{n}_\mathbf{y}} \sigma(\mathbf{y})\, dA_\mathbf{y} - \int_\Gamma G(\mathbf{x},\mathbf{y}) \tau(\mathbf{y})\, dA_\mathbf{y}$$

Double-layer potential — Single-layer potential

$[u(\mathbf{x})]_\Gamma = 0$ — $[u(\mathbf{x})]_\Gamma = \sigma(\mathbf{x})$ — $u(\mathbf{x})|_{\mathbf{x}\in\mathbb{R}^d\backslash\Omega} = 0$



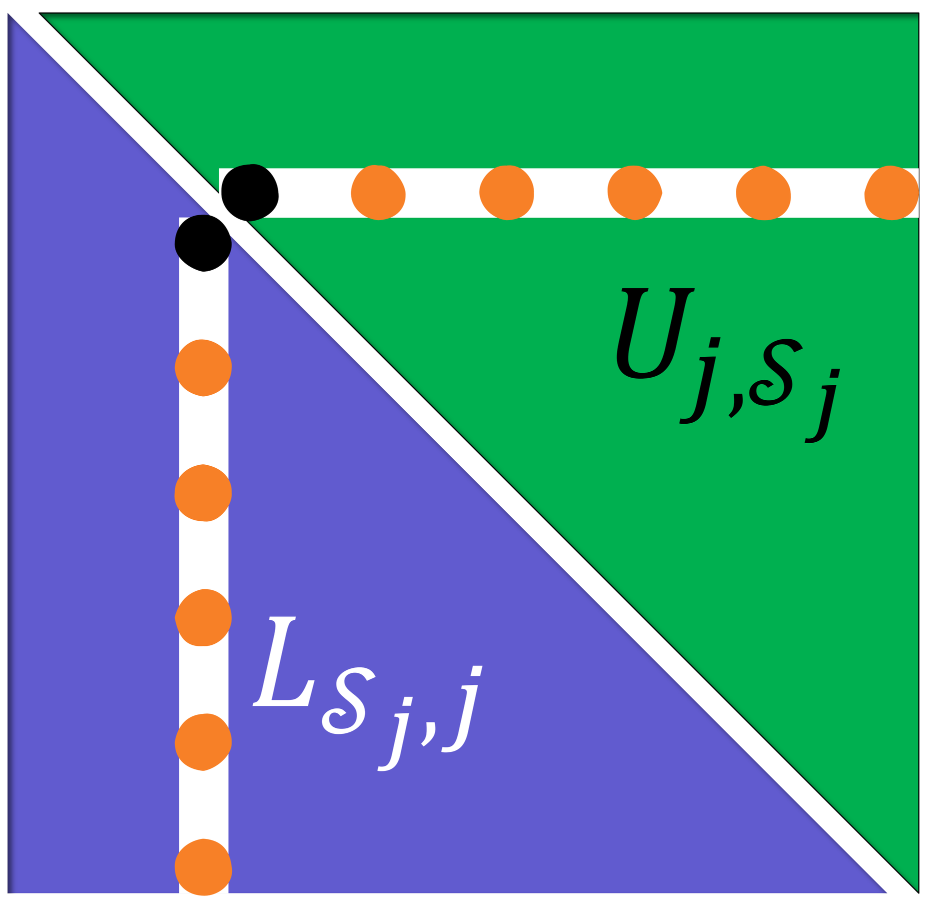
Variants of Boundary Integral Equations

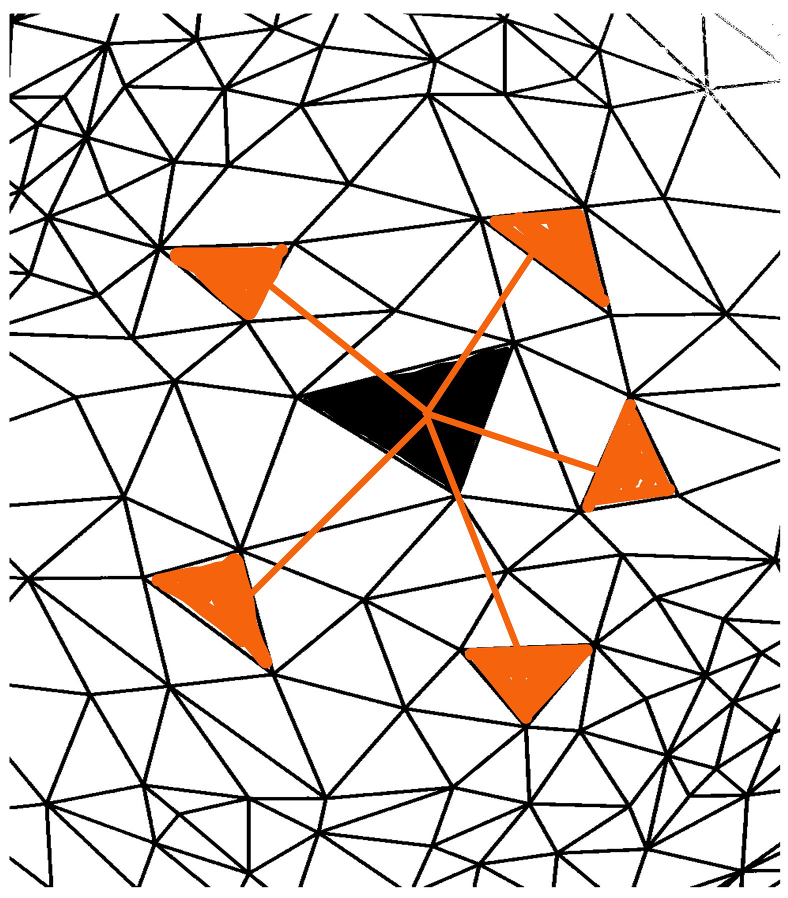
How our inverse preconditioner works

$$Kx = b \Rightarrow K^{-1} \approx L_S U_S \Rightarrow x \approx L_S U_S b$$

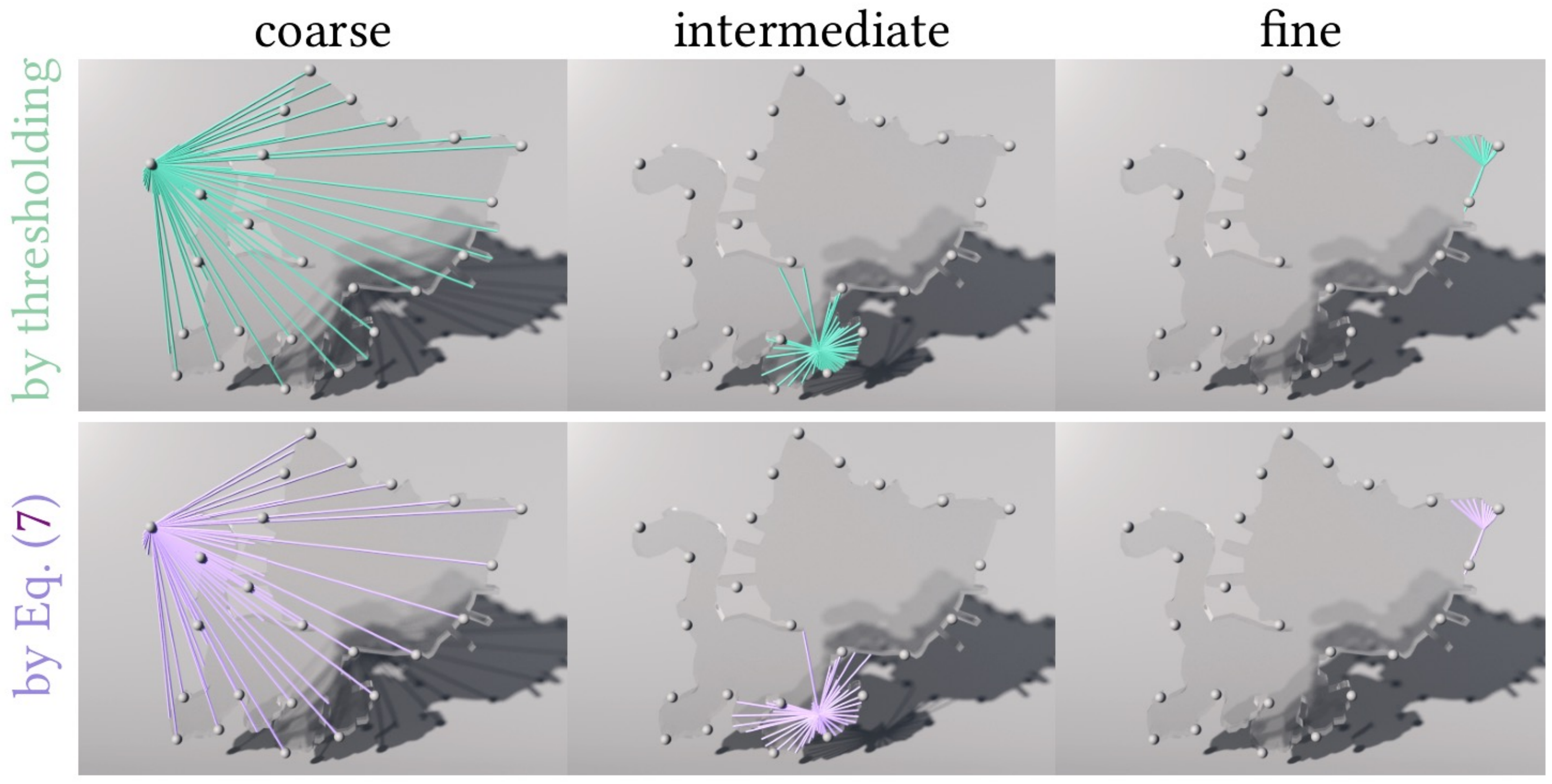
Extended Kaporin's construction

$$\mathbf{L}_{s_j,j} = \frac{\mathbf{G}^{-1}_{s_j,s_j} \mathbb{e}_j}{\mathbb{e}_j^\top \mathbf{G}^{-1}_{s_j,s_j} \mathbb{e}_j} \qquad \mathbf{U}^\top_{j,s_j} = \mathbf{G}^{-\top}_{s_j,s_j} \mathbb{e}_j$$

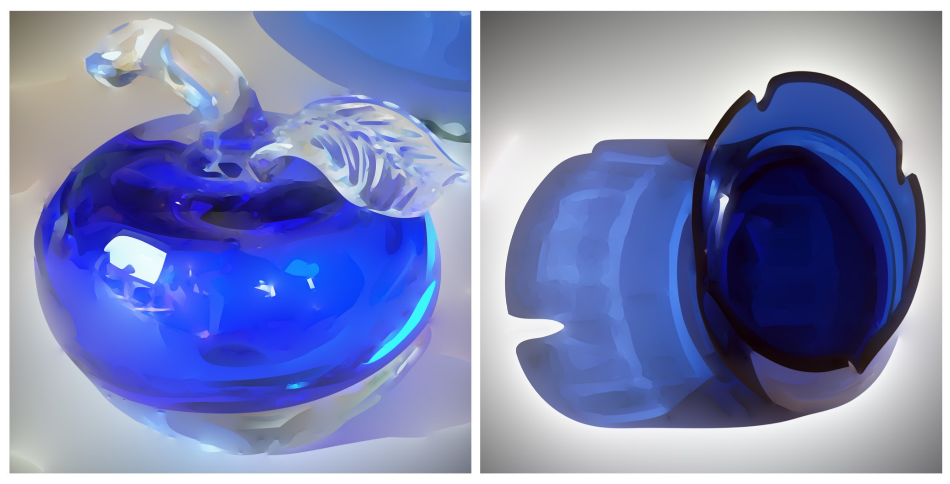


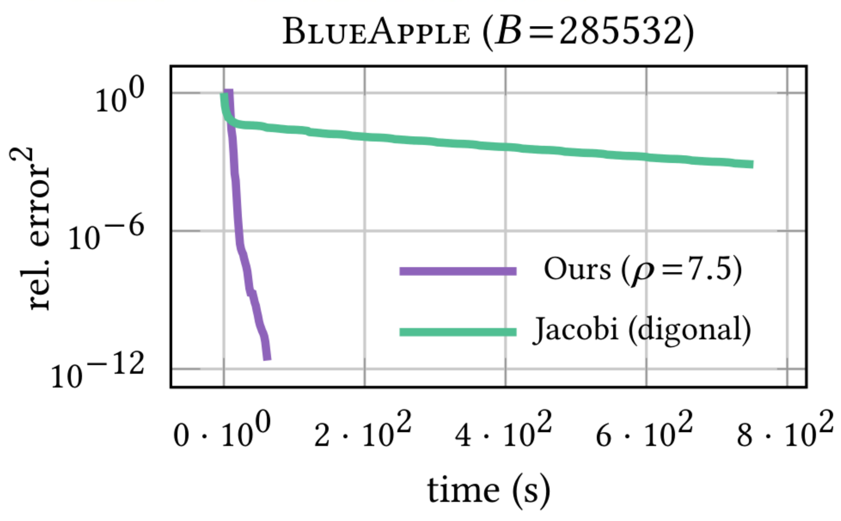


Ground-truth vs. max-min sparsity

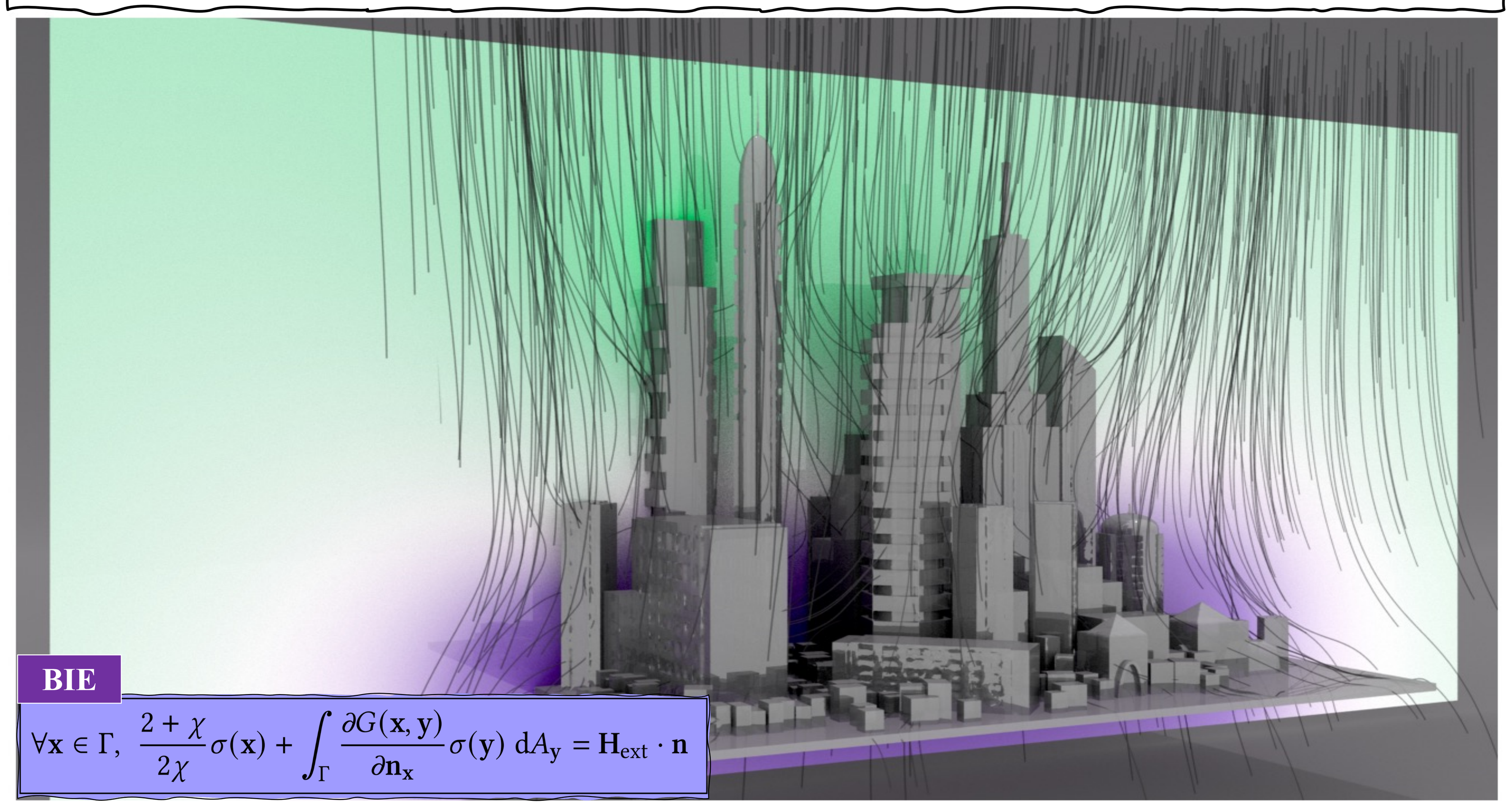


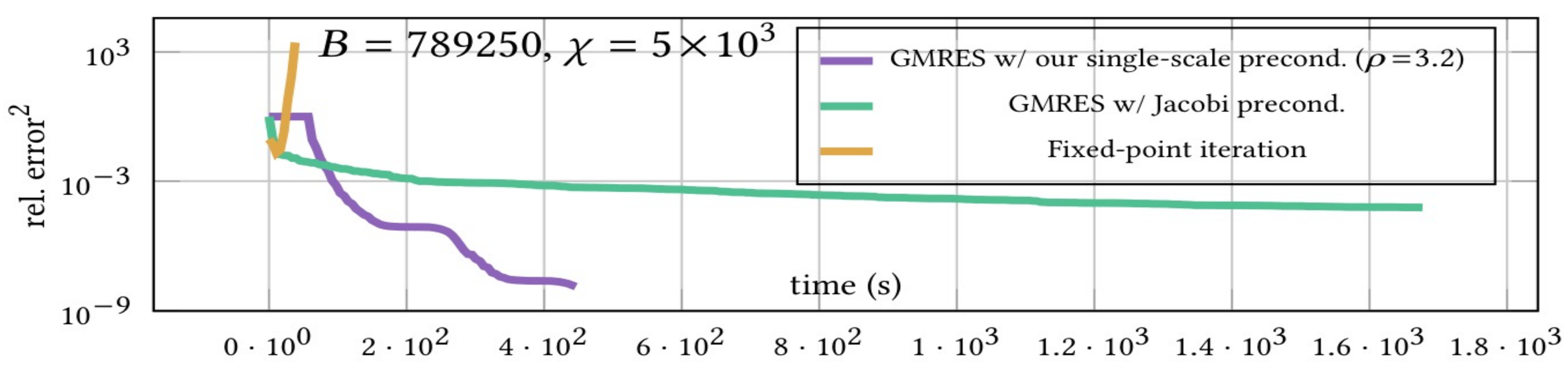
Dirichlet boundary condition



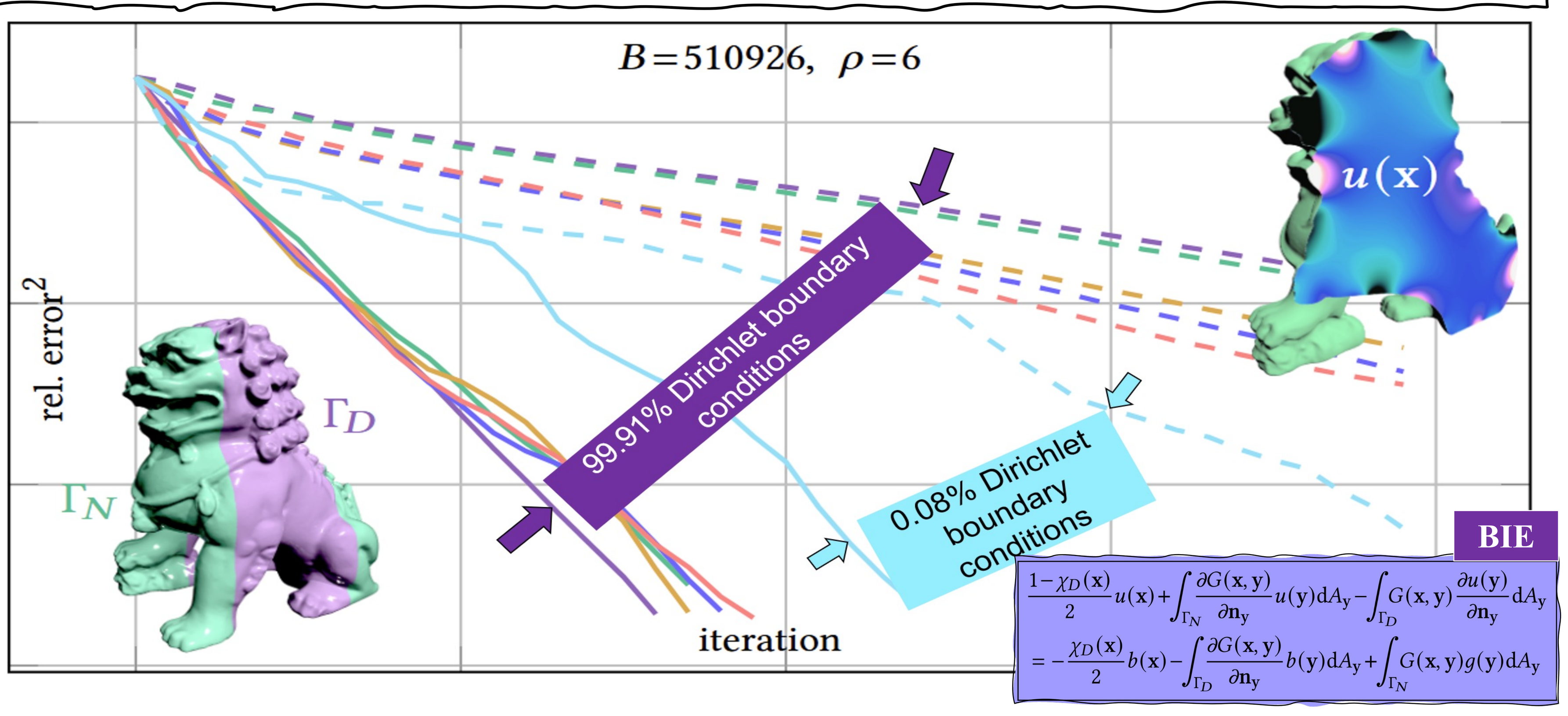


Neumann boundary condition





Mixed boundary condition



Take-home on the choice of BEM formulation