SIC Saarland Informatics Campus

SAARBRÜCKEN

EUROGRAPHICS 2023

Robust Pointset Denoising of Piecewise-Smooth Surfaces through Line Processes

Jiayi Wei[1,2], Jiong Chen[1], Damien Rohmer[1], Pooran Memari[1,2], Mathieu Desbrun[2,1]

[1]LIX, Ecole Polytechnique/CNRS, IP Paris, France [2]INRIA Saclay, France

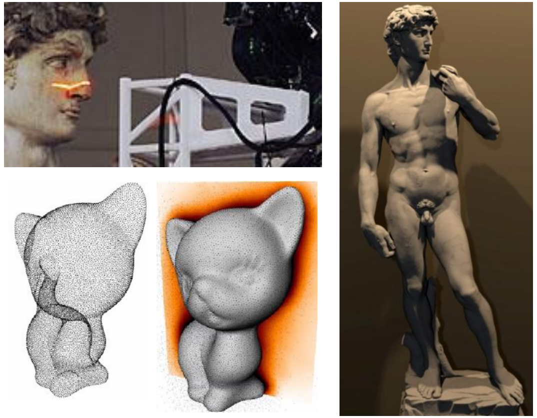
SIC Saarland Informatics Campus

SAARBRÜCKEN

EUROGRAPHICS 2023

Background

Background



Scanning is ubiquitous and paramount

- from medical and commerce applications to art preservation

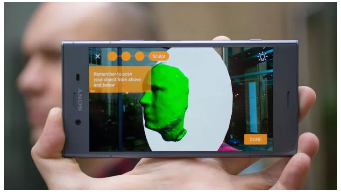
Denoising the raw output pointset and meshing it are well studied

- IF ONLY little noise and very few outliers

But… cheaper sensors, aggregated data = noisier, outlier-ridden pointsets!

Konica Minolta Vivid 9i
~$10K in 2004

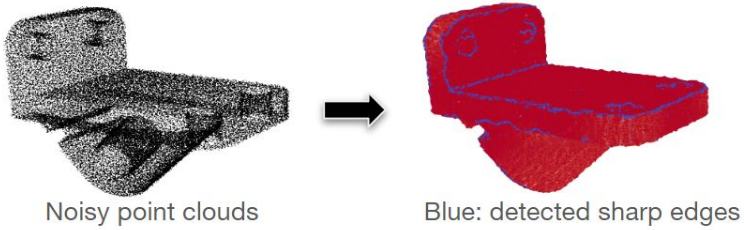
~$150 in 2021



Technical Challenges

- Challenge 1: Being agnostic to noise models

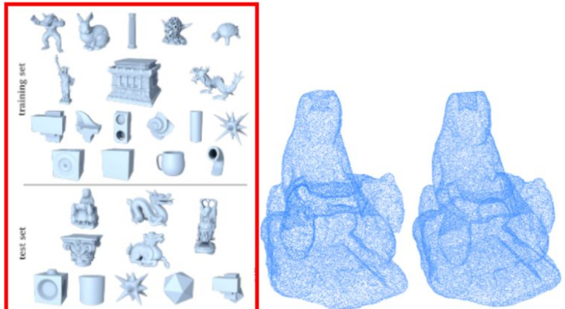
- Challenge 2: Identifying & preserving sharp features



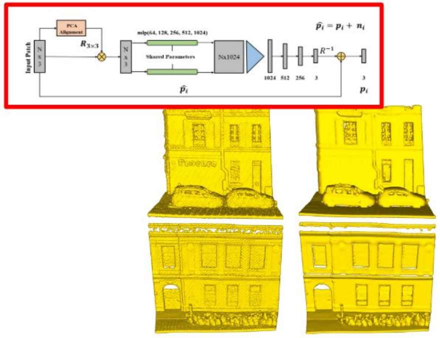
Previous Work
Learning-based Methods

Advantage:
- Very good performance for specific sensors

Disadvantage:
- Usually fail for other sensors or merged data



cleaned with regularity term   ground truth



(a) Noisy input   (e) Ours

Learning-based methods
[Rakotosaona19,Zhang21]

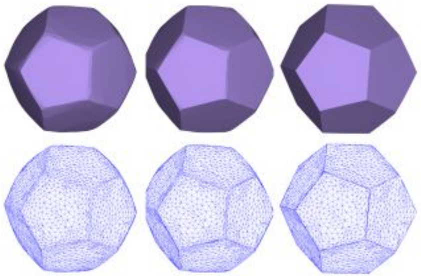
Previous Work
Optimization-based Methods

Advantage:

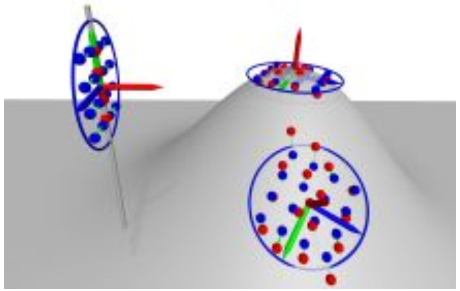
- Work well for specific applications

Disadvantage:

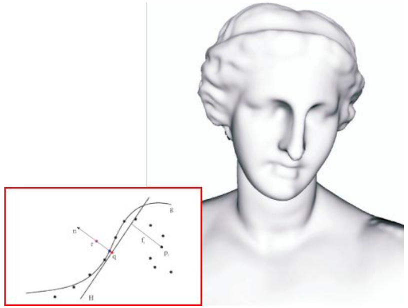
- Require prior knowledge on the inputs
- High computational cost
- Suffer over-smoothing / over-sharpening



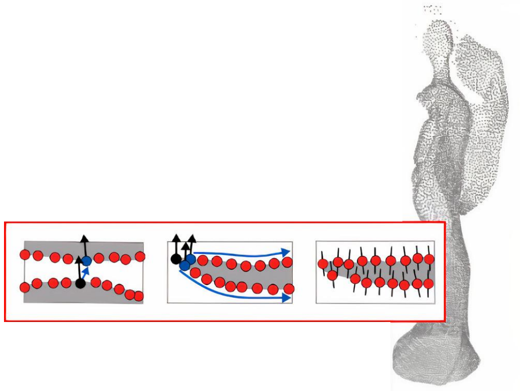
ℓ_0 minimization [He13]



non-local dictionary-based method [Digne et al. 18]



MLS (moving least squares)-based methods
[Alexa at al. 03, Guennebaud et al. 07]

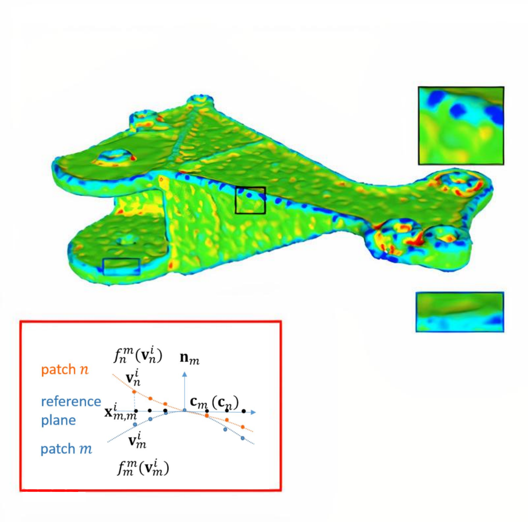


LOP (locally optimal projection)-based methods
[Huang et al. 09, Liao et al. 13]

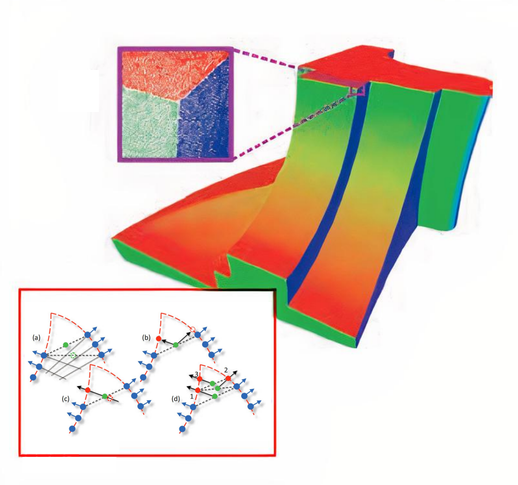
Previous Work
Filtering-based Methods

Advantage:
- Least amount of assumption on the input data

Disadvantage:
- Pre-canned formula
- No control over filtering except for #iterations



Spectral-based methods [Digne et al. 2012, Zeng et al. 2019]



Normal filtering [Huang et al. 2013, Digne et al. 2017]

SIC Saarland Informatics Campus

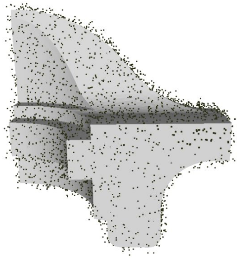
SAARBRÜCKEN

EUROGRAPHICS 2023

Method

Mathematical Background

Robust Statistics & Line Process (Outlier Process)



Fandisk model and its noisy point cloud

Goals of Robust Statistics

- "Describe the structure best fitting the bulk of the data";
- "identify the deviating data points (outliers) for further treatment"

Objective Function:

data measurement

$$\min_a \sum_{s \in S} \rho(d_s - u(s;a))$$

error metric / rho unction

Fit of model

[Black et al. 1996]

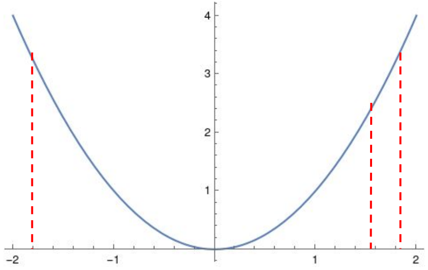
Residual error

penalty function

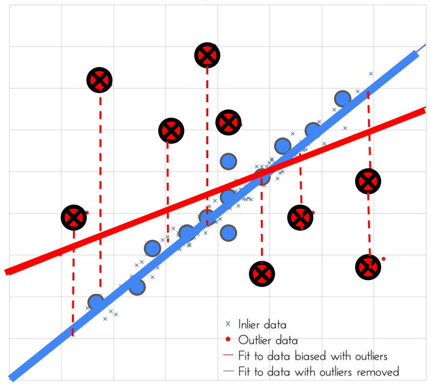
$$\min_{a,z} \sum_{s \in S} (d_s - u(s;a))^2 z + \Psi(z)$$

line process

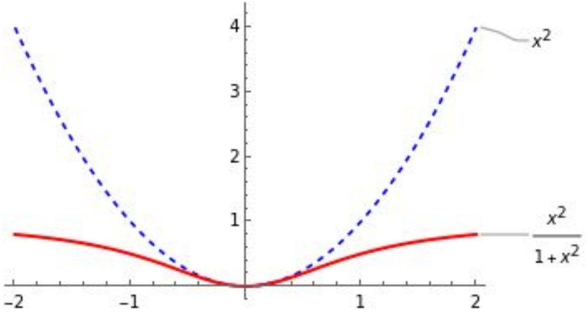
0 <= z <= 1



$\rho(x) = x^2$



E.g. [Geman,McClure87]:



ρ-function $\rho(x) = \frac{x^2}{1+x^2}$

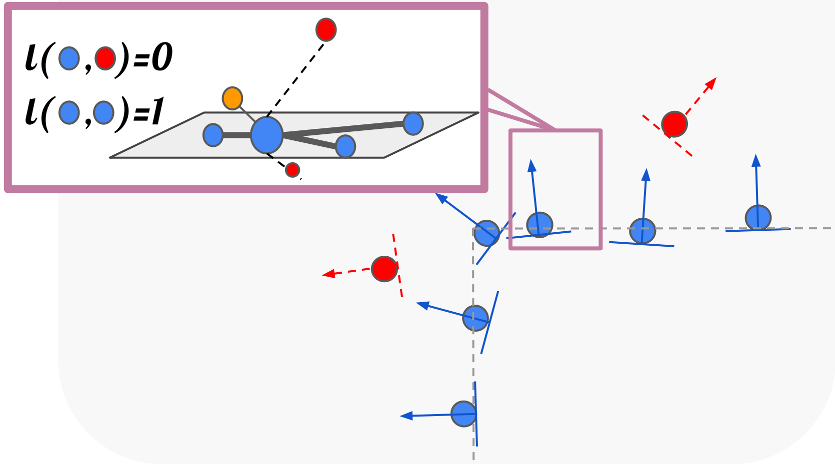
Our approach at a glance

Goal: a *variational* approach

- Least amount of assumption on inputs
- balance between **local fit** and **global smoothness**
- convenience of ***line processes*** for robustness and computational efficiency

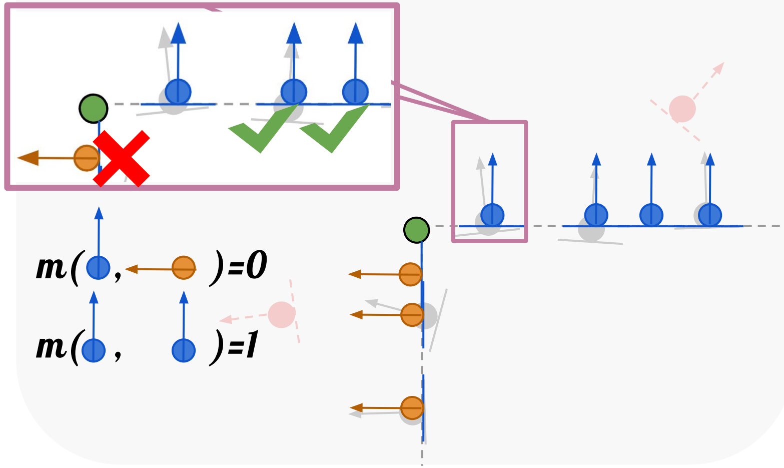
Local (tangent plane) fitting:

- Line process (**l**$_{i}$□) helps to reject **outliers**



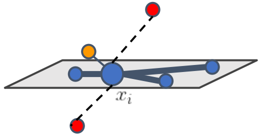
Global piecewise smoothness:

- Line process (**m**$_{i}$□) helps with **feature preservation**

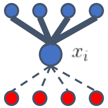


Formulation - First Trial

line process parameter m: favors the inliers that fit the plane



line process parameter l: help preserve sharp features



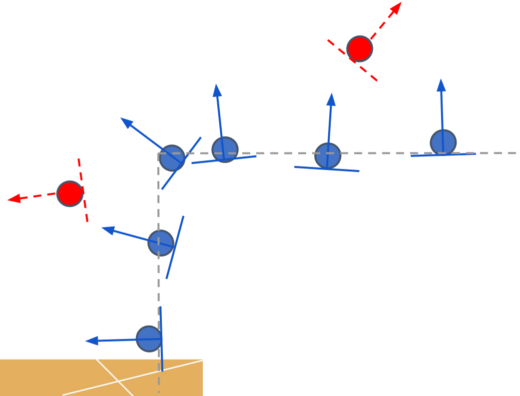
$$\min_{c} E = E_{local} + \lambda E_{global}$$

$$= \sum_{patch\ i} \sum_{j \in \mathcal{N}(i)} \left[\mathbf{c}_i \mathbf{P}_j \mathbf{P}_j^\intercal \mathbf{c}_i^\intercal m_{ij} + \Psi(m_{ij}) \right] + \lambda \sum_{i,\, j \in \mathcal{E}} \left[w_{ij} ||\mathbf{c}_i - \mathbf{c}_j||^2 l_{ij} + \Psi(l_{ij}) \right]$$

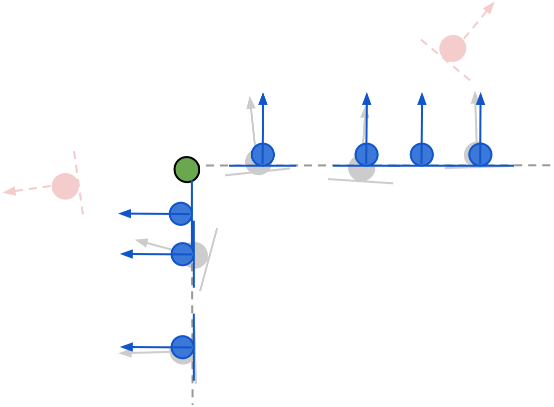
$$\mathbf{c} = \begin{bmatrix} \mathbf{n} \\ d \end{bmatrix}$$

$$\mathbf{c}^\intercal x = 0$$

patch fitting



smoothness of patch stitching



Formulation - Other trials

$$\min_{c} E = E_{local} + \lambda E_{global}$$

$$= \sum_{patch\ i} \sum_{j \in \mathcal{N}(i)} \left[\mathbf{c}_i \mathbf{P}_j \mathbf{P}_j^\intercal \mathbf{c}_i^\intercal m_{ij} + \Psi(m_{ij}) \right] + \lambda \sum_{i,j \in \mathcal{E}} \left[w_{ij} ||\mathbf{c}_i - \mathbf{c}_j||^2 l_{ij} + \Psi(l_{ij}) \right]$$

local fitting

global smoothing

Has degenerated solution: c = 0

Add a constraint:

$$||\mathbf{c}_i|| = 1$$

Formulation - Other trials

$$\min_{c} E = E_{local} + \lambda E_{global}$$

$$= \sum_{patch\ i} \sum_{j \in \mathcal{N}(i)} \left[\mathbf{c}_i \mathbf{P}_j \mathbf{P}_j^\intercal \mathbf{c}_i^\intercal m_{ij} + \Psi(m_{ij}) \right] + \lambda \sum_{i,j \in \mathcal{E}} \left[w_{ij} ||\mathbf{c}_i - \mathbf{c}_j||^2 l_{ij} + \Psi(l_{ij}) \right]$$

local fitting

global smoothing

Has degenerated solution: c = 0

Add a constraint:

$$||\mathbf{c}_i|| = 1$$

Increases complexity

$$\min_{\tilde{c}} E = \sum_{patch\ i} \sum_{j \in \mathcal{N}(i)} \left[\tilde{c}_i \mathbf{P}_j \mathbf{P}_j^\intercal \tilde{c}_i^\intercal m_{ij} + \Psi(m_{ij}) \right]$$

$$\min_{c} E = \sum_{patch\ i} (||\mathbf{c}_i - \tilde{\mathbf{c}}_\mathbf{i}||^2) + \lambda \sum_{i,j \in \mathcal{E}} (||\mathbf{c}_i - \mathbf{c}_j||^2 w_{ij} l_{ij} + \Psi(l_{ij}))$$

Formulation - Other trials

$$\min_{c} E = E_{local} + \lambda E_{global}$$

$$= \sum_{patch\ i} \sum_{j \in \mathcal{N}(i)} \left[\mathbf{c}_i \mathbf{P}_j \mathbf{P}_j^\intercal \mathbf{c}_i^\intercal m_{ij} + \Psi(m_{ij}) \right] + \lambda \sum_{i,\, j \in \mathcal{E}} \left[w_{ij} ||\mathbf{c}_i - \mathbf{c}_j||^2 l_{ij} + \Psi(l_{ij}) \right]$$

local fitting

global smoothing

Has degenerated solution: c = 0

Add a constraint:

$$||\mathbf{c}_i|| = 1$$

Increases complexity

$$\min_{\tilde{c}} E = \sum_{patch\ i} \sum_{j \in \mathcal{N}(i)} \left[\tilde{c}_i \mathbf{P}_j \mathbf{P}_j^\intercal \tilde{c}_i^\intercal m_{ij} + \Psi(m_{ij}) \right]$$

$$\min_{c} E = \sum_{patch\ i} (||\mathbf{c}_i - \tilde{\mathbf{c}}_i||^2) + \lambda \sum_{i,\, j \in \mathcal{E}} (||\mathbf{c}_i - \mathbf{c}_j||^2 w_{ij} l_{ij} + \Psi(l_{ij}))$$

Relies too much on the initial guess of c

Introduce **slack variables** between the two stages

Final Formulation

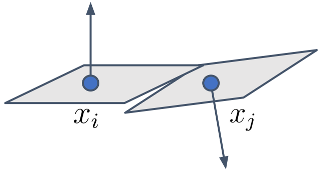
$$\min_{\mathbf{H},\mathbf{T},\mathbb{L},\mathbb{M},\mathbb{S}} \underbrace{\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in N(i)\cup\{i\}} \alpha_i \left[\mathbf{h}_i^t\, \bar{\mathbf{q}}_j \bar{\mathbf{q}}_j^t\, \mathbf{h}_i\, l_{ij} + \Psi_{\mu_l}(l_{ij})\right]}_{\text{local fitting}}$$

$$+ \underbrace{\frac{\lambda}{2}\sum_{(i,j)\in M} \beta_{ij} \left[||\mathbf{t}_i - s_{ij}\mathbf{t}_j||^2 m_{ij} + \Psi_{\mu_m}(m_{ij})\right]}_{\text{piecewise smoothness}}$$

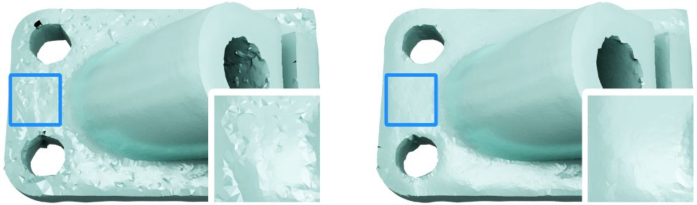
$$+ \underbrace{\frac{\eta}{2}\sum_{i=1}^{n} \alpha_i ||\mathbf{h}_i - \mathbf{t}_i||^2}_{\text{stitching}} \qquad \|\mathbf{h}_i\| = 1\ \forall i \in [1, n].$$

$$\mathbf{h} = \begin{bmatrix}\mathbf{n}\\ d\end{bmatrix} \qquad \mathbf{t} = \begin{bmatrix}\mathbf{n}\\ d\end{bmatrix}$$

$$h^T q = 0 \qquad t^T q = 0$$



normal orientation problem



without s with s

Final Formulation

$$\min_{\mathbf{H},\mathbf{T},\mathbb{L},\mathbb{M},\mathbb{S}} \quad \underbrace{\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in N(i)\cup\{i\}} \alpha_i \left[\mathbf{h}_i^t\, \bar{\mathbf{q}}_j \bar{\mathbf{q}}_j^t\, \mathbf{h}_i\, l_{ij} + \Psi_{\mu_l}(l_{ij})\right]}_{\text{local fitting}}$$

$$+ \underbrace{\frac{\lambda}{2}\sum_{(i,j)\in M} \beta_{ij}\left[||\mathbf{t}_i - s_{ij}\,\mathbf{t}_j||^2\, m_{ij} + \Psi_{\mu_m}(m_{ij})\right]}_{\text{piecewise smoothness}}$$

$$+ \underbrace{\frac{\eta}{2}\sum_{i=1}^{n} \alpha_i ||\mathbf{h}_i - \mathbf{t}_i||^2}_{\text{stitching}} \qquad \|\mathbf{h}_i\| = 1 \;\forall i \in [1, n].$$

$$\mathbf{h} = \begin{bmatrix}\mathbf{n}\\ d\end{bmatrix} \qquad h^T q = 0$$

≅

$$\mathbf{t} = \begin{bmatrix}\mathbf{n}\\ d\end{bmatrix} \qquad t^T q = 0$$

Advantages:

- **Least amount of assumptions** on the inputs
- **Robust** local estimates, feature preservation
- Easy to optimize (**block-coordinate descent**)
- Good convergence from arbitrary initializations (in practice)

Optimization in Practice

$$\min_{\mathbf{H},\mathbf{T},\mathbb{L},\mathbb{M},\mathbb{S}} \underbrace{\frac{1}{2}\sum_{i=1}^{n}\sum_{j\in N(i)\cup\{i\}} \alpha_i \left[\mathbf{h}_i^t\, \bar{\mathbf{q}}_j \bar{\mathbf{q}}_j^t\, \mathbf{h}_i\, l_{ij} + \Psi_{\mu_l}(l_{ij})\right]}_{\text{local fitting}}$$

$$+ \underbrace{\frac{\lambda}{2}\sum_{(i,j)\in M} \beta_{ij}\left[||\mathbf{t}_i - s_{ij}\,\mathbf{t}_j||^2\, m_{ij} + \Psi_{\mu_m}(m_{ij})\right]}_{\text{piecewise smoothness}}$$

$$+ \underbrace{\frac{\eta}{2}\sum_{i=1}^{n} \alpha_i ||\mathbf{h}_i - \mathbf{t}_i||^2}_{\text{stitching}} \qquad \|\mathbf{h}_i\| = 1\ \forall i \in [1, n].$$

Key idea: ***block-coordinate descent*** method

Local fitting

- Update local tangent plane ***H***
  - ≈ solve **eigenvalue problems** of size 4x4 in parallel
- Update local fitting line process ***L***
  - **closed-form solution** related to μ_l, ***h***

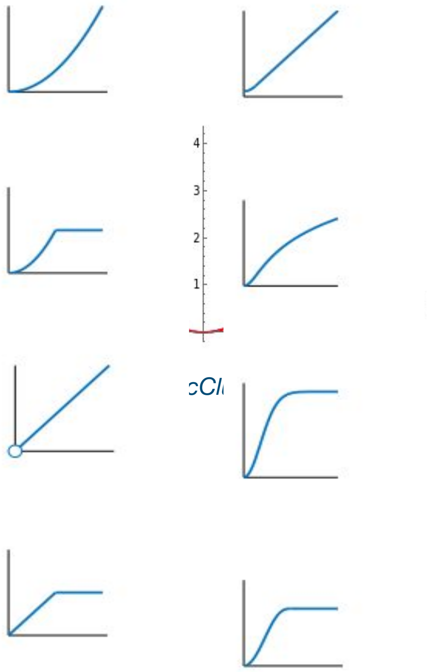
Piecewise smoothness

- Update tangent plane ***T***
  - = solve **a sparse SPD linear system** (symmetric positive definite)
- Update piecewise smoothness line process ***M***
  - **closed-form solution** related to μ_m, ***t***
- Update normal-flipping variable ***S***
  - **closed-form solution** related to ***t***

Summary of Our Approach

- Generic tool in the denoising toolbox based on robust estimator
  - other examples: the family of nonlinear normal filters [Yadav et al. 2020]

- From **robust estimator** to **line process**
  - non-linear optimization -> block-descent optimization
  - detects outliers / sharp features
  - fast and stable



Common M-estimators [Yadav et al. 2020]

SIC Saarland Informatics Campus

SAARBRÜCKEN

EUROGRAPHICS 2023

Results

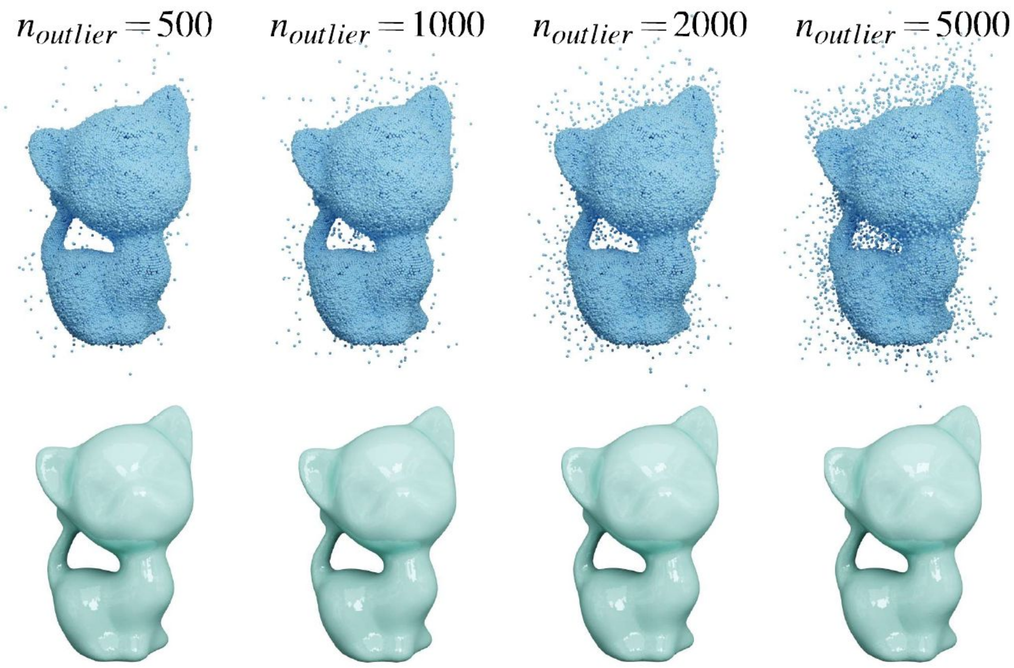
Results: Robustness to outliers

Input:

- Noisy kitten model (~25k points) with manually added outliers

Output:

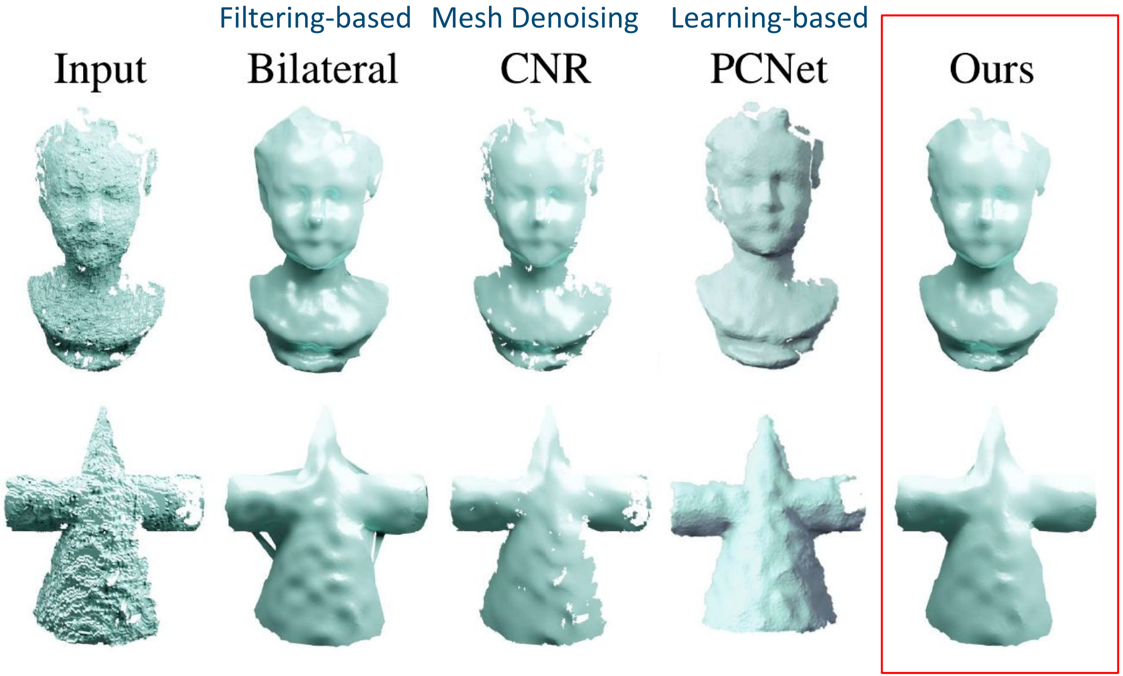
- Denoised pointsets
- Surface reconstruction for visual purposes



Results: Robustness to real data

Input:

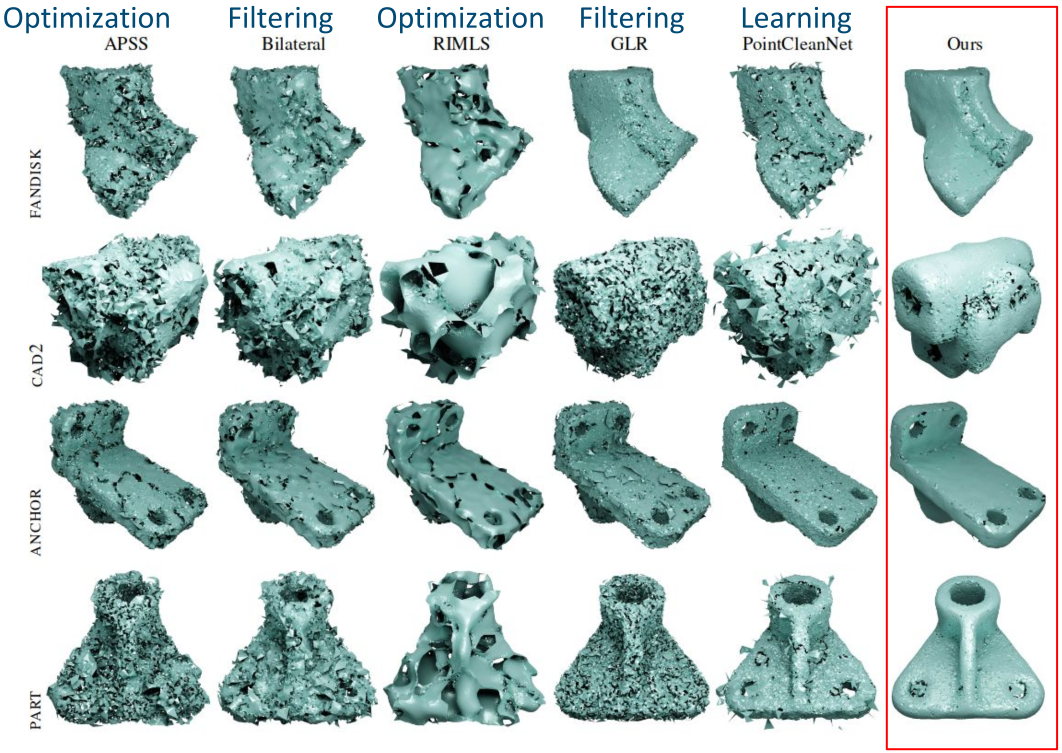
- Real scanned data (mesh) from Kinect



Results: Robustness to Gaussian Noise (stress test)

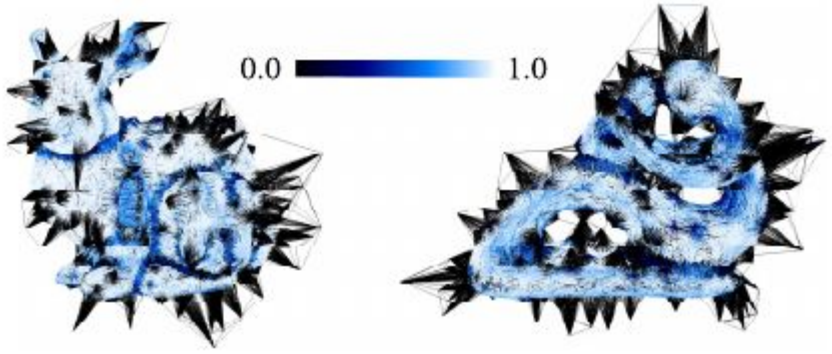
Input:

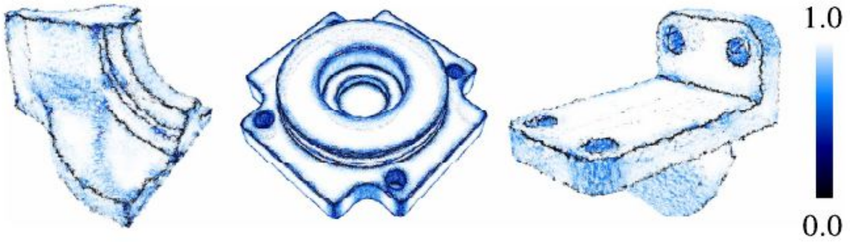
- CAD models with a **very large noise level** (Gaussian noise with variance 3% of bounding box)



Results: Visualizing Outliers and Features

- **Outlier Indicator** via ***l*** (local fitting)
  - Color-coded line segment = min(l_{ij},l_{ji})
- **feature indicator** via ***m*** (global smoothness)

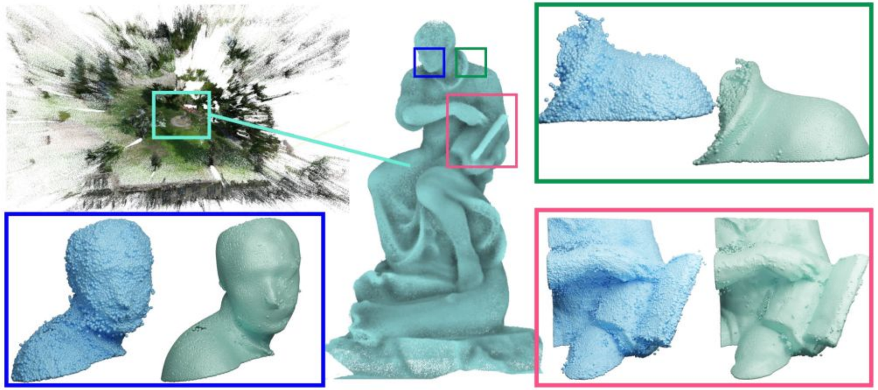




Results: Performance

Large-scale denoising

1.15 million pts, ~115s



Linear solver:
Multigrid-preconditioned conjugate method (amg-pcg)

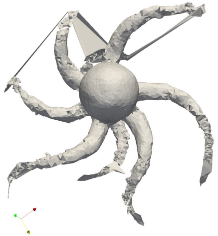
Comparison with other methods

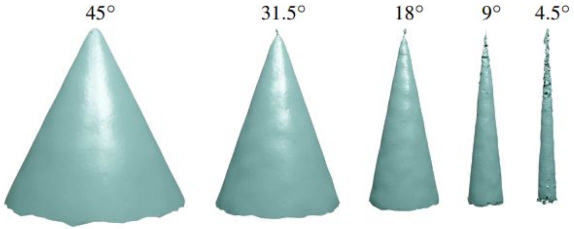
Accessory: 50K points

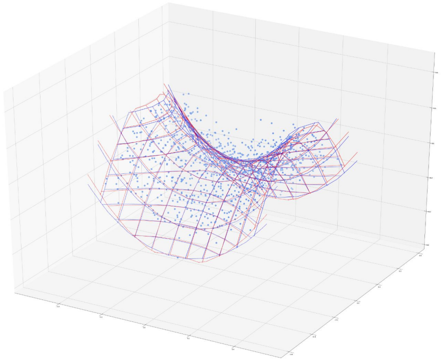
Methods	Bilateral	APSS	RIMLS	GLR	PointCleanNet	Ours
RunTime	13.31s	19.05s	35.45s	400s	438s	**16.21s**

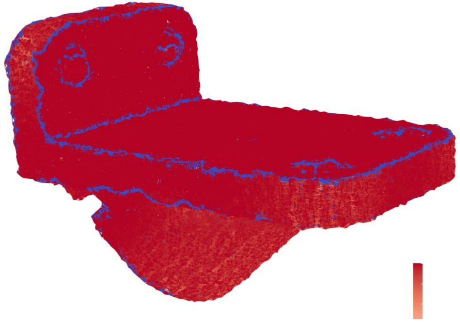
Limitations and Future Work

- **Dealing with thin structure**
  - adaptive selectivity of line process
- **Restricted to low-order fitting**
  - quadric fitting?
- **Better post-treatment of detected sharp features**

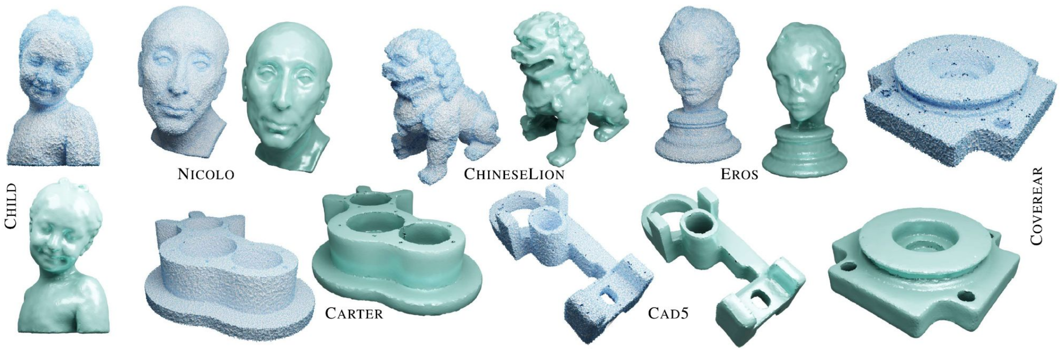








Thank you!

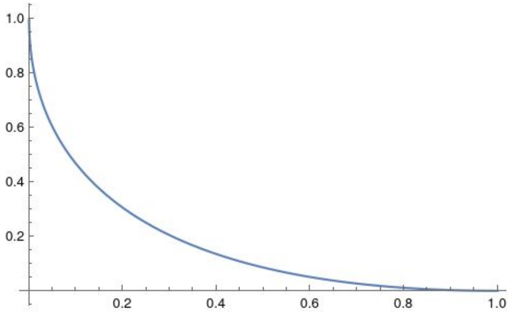


Complementary: Penalty function

[Black1996]: Black, Michael J., and Anand Rangarajan. "On the unification of line processes, outlier rejection, and robust statistics with applications in early vision." *International journal of computer vision* 19.1 (1996): 57-91.

penalty function

$$\min_{z} \sum_{s \in S} (d_s - u(s))^2 z + \boxed{\Psi(z)}$$



$$\Psi(z) = \mu(1 - \sqrt{z})^2$$

Complementary: Effects of parameters

Relies on 3 main parameters:

- Selectivity μ_m and μ_l
- Smoothness strength λ